

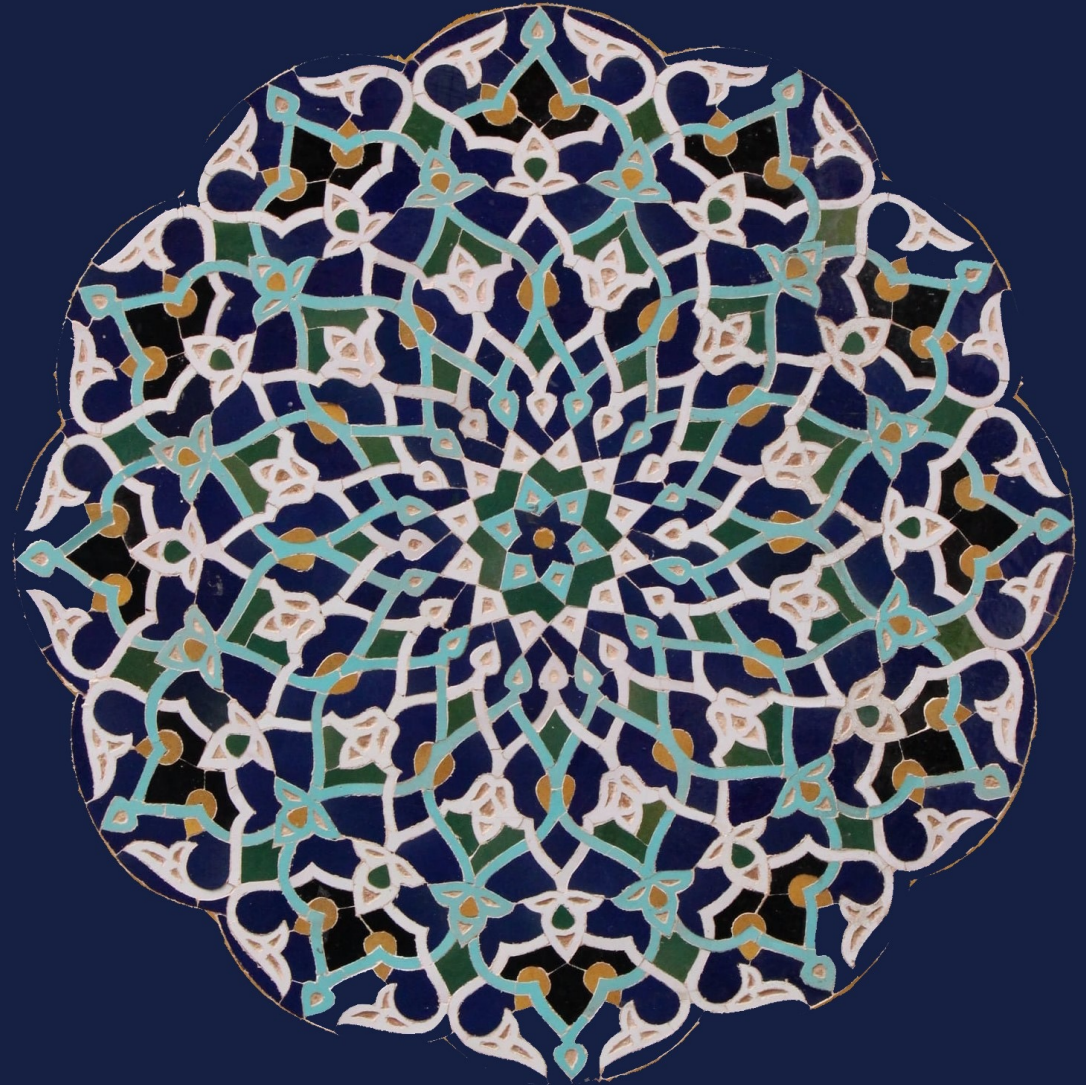


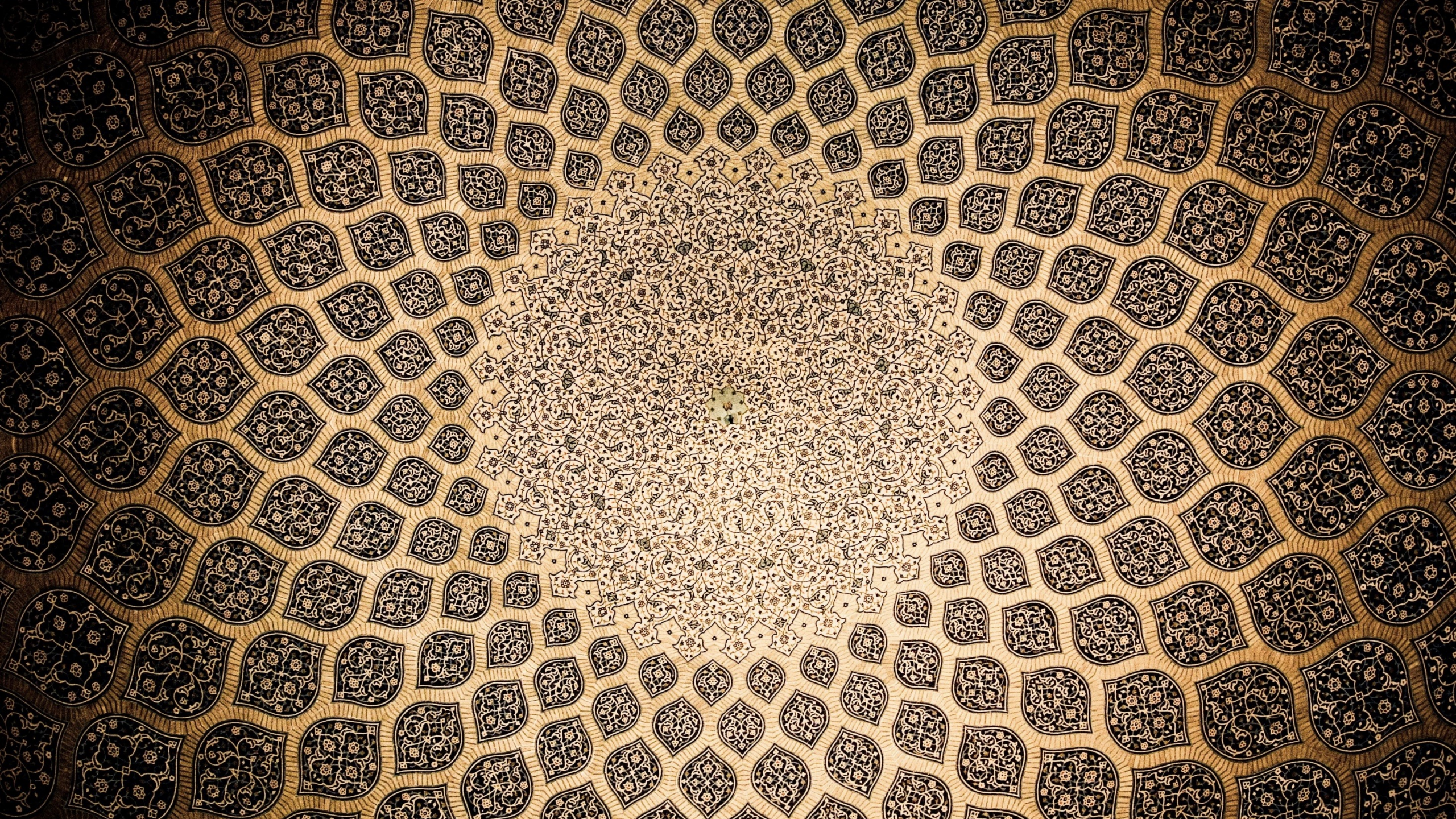
GRESHAM
COLLEGE

The Mathematics of Beauty and Symmetry

Professor Sarah Hart

Gresham Professor of Geometry



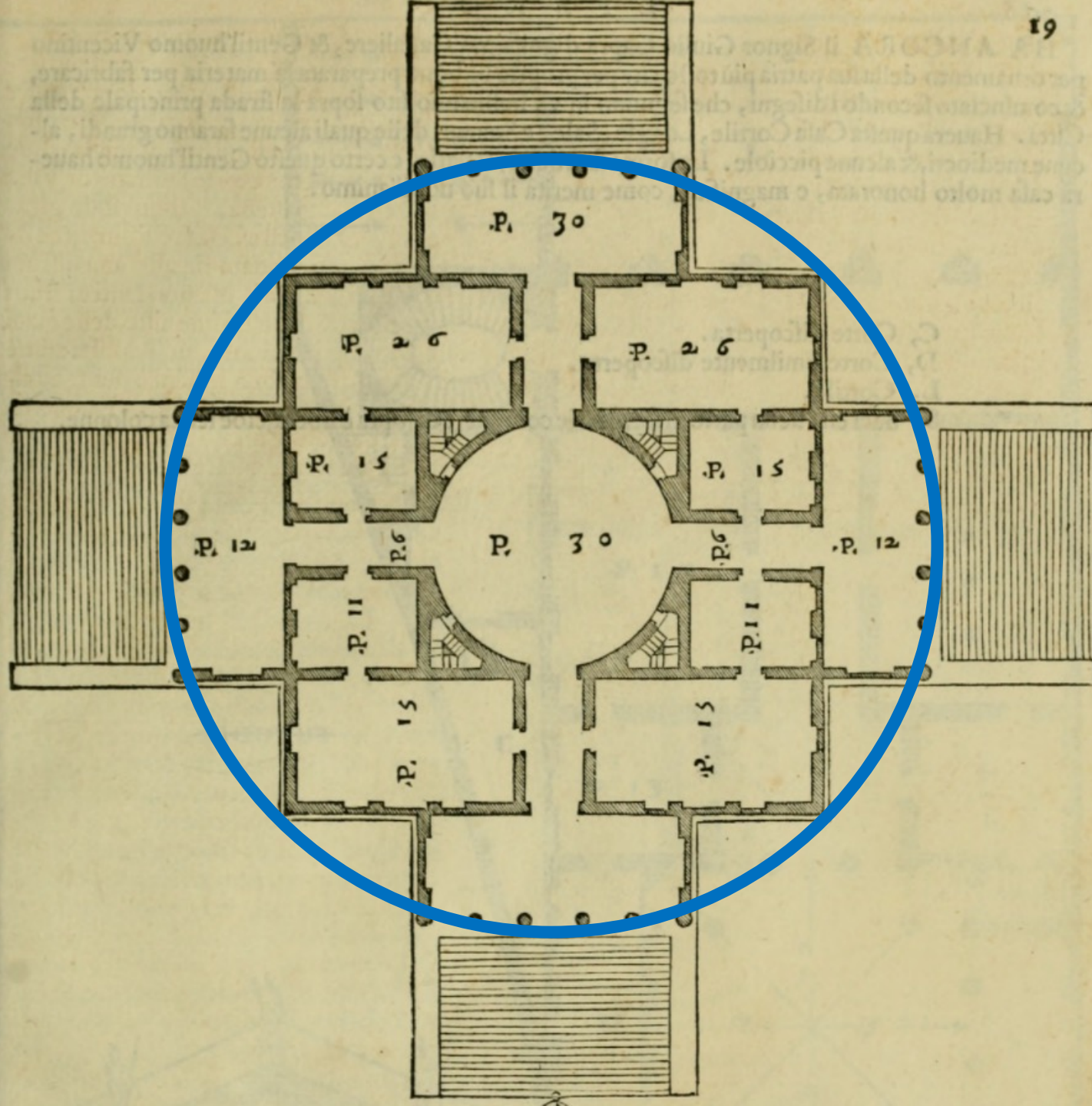








Villa Rotonda: Image credit Marco Bagarella

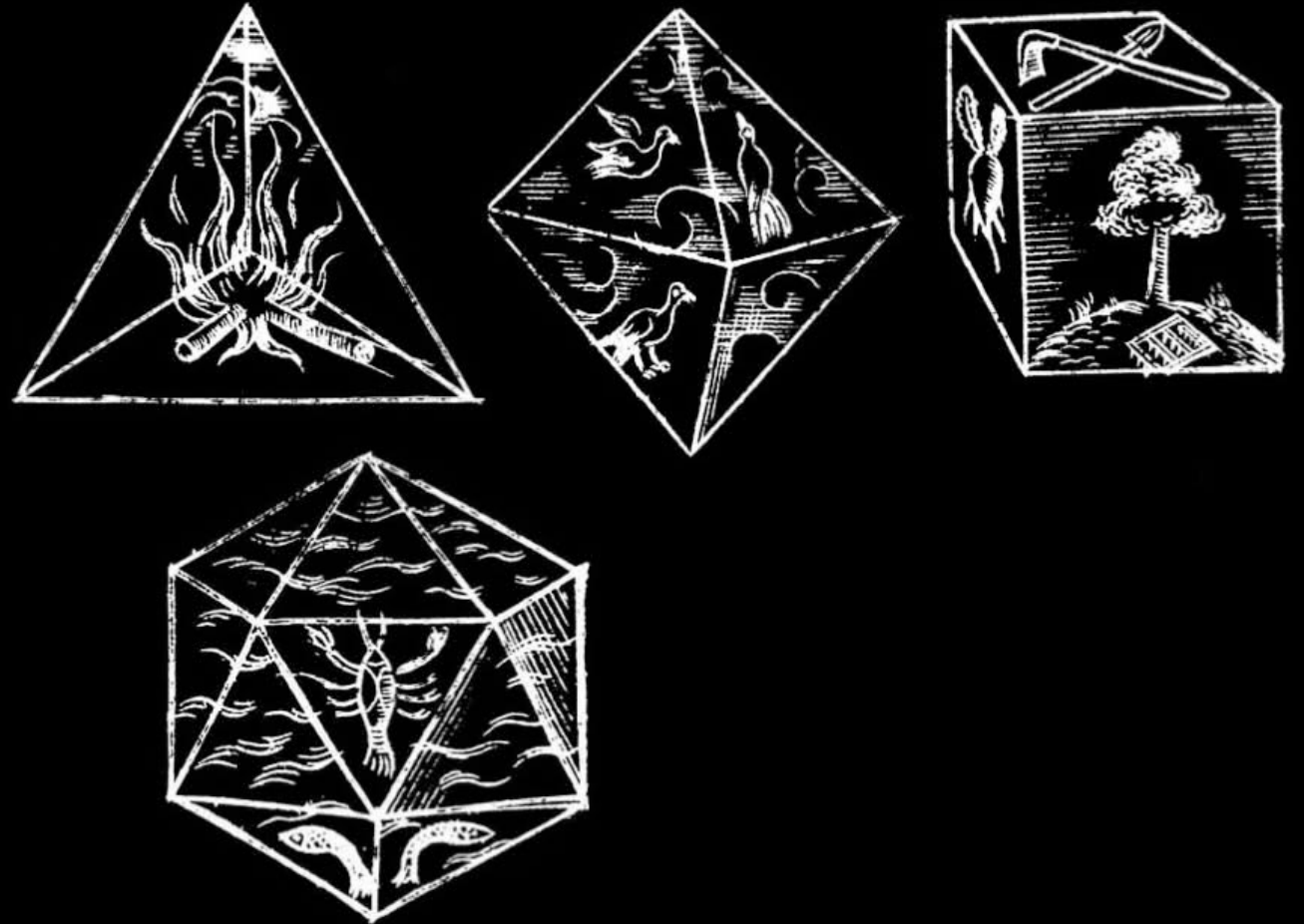


Why do we like symmetry?



The “most beautiful figures”

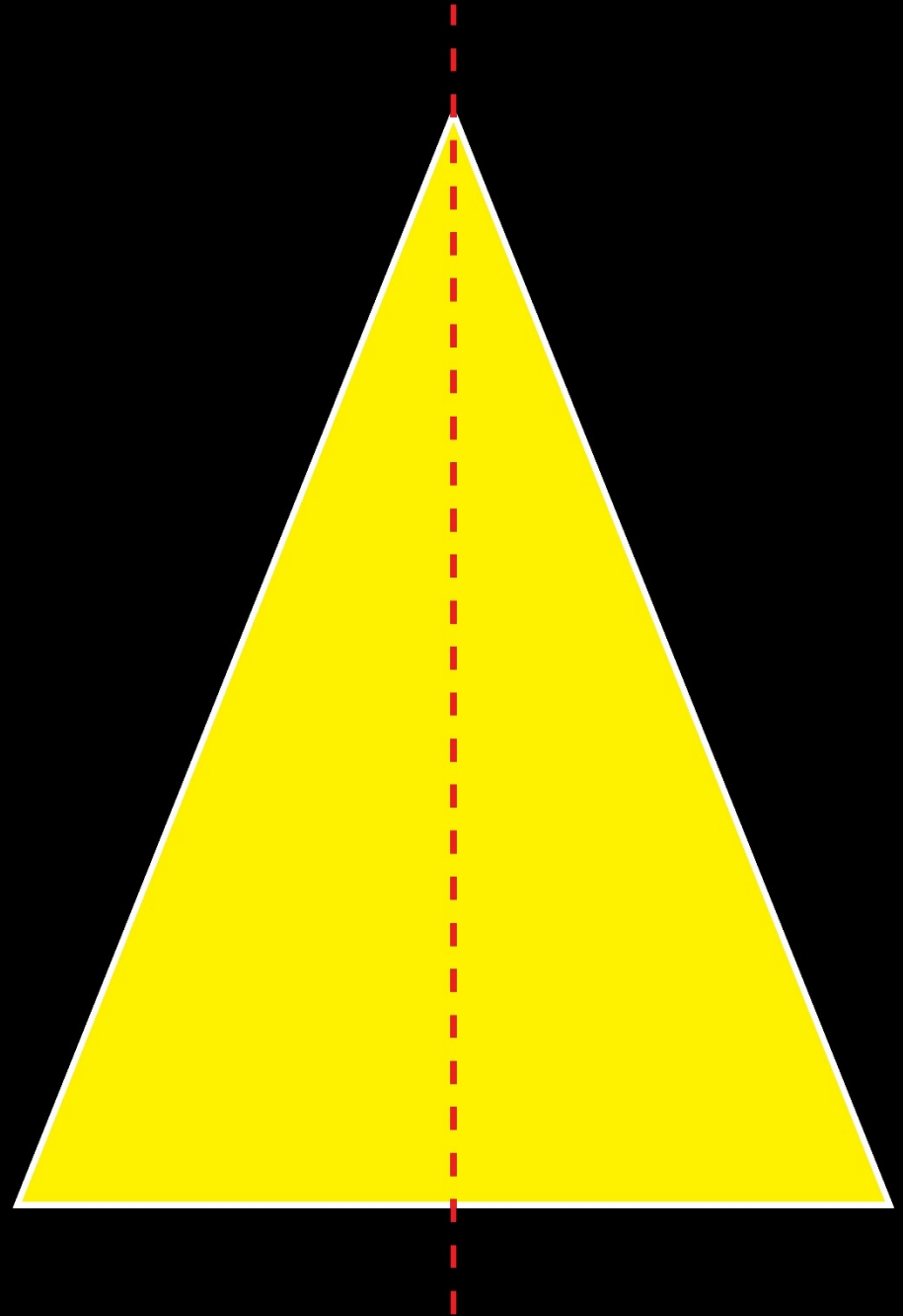
In the *Timaeus* (c 360 B.C.) Plato explains the creation of the world, by “*a method with which your scientific training will have made you familiar. Fire, air, earth, and water are bodies and therefore solids [...] we must determine what are the four most beautiful figures.*”



“There is a fifth figure (which is made out of 12 pentagons), the dodecahedron – this God used as a model for the twelvefold division of the Zodiac.”

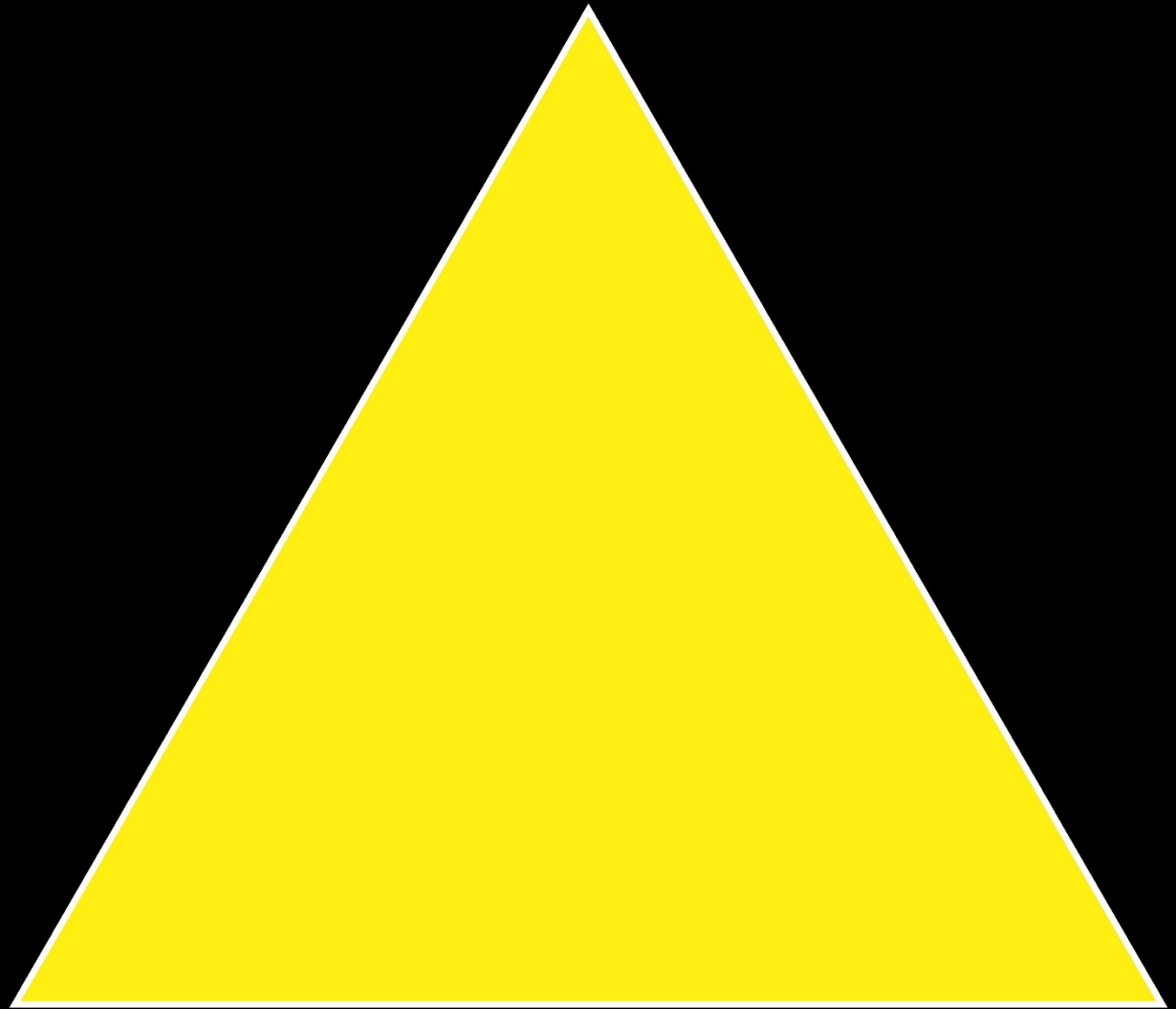
Measuring Symmetry

- The symmetries of an object are the “things you can do to it that leave it looking the same”.
- (More accurately, distance-preserving transformations that map the object to itself)
- Isosceles Triangle
 - 2 symmetries
 - Reflection
 - “do nothing” (identity map I)

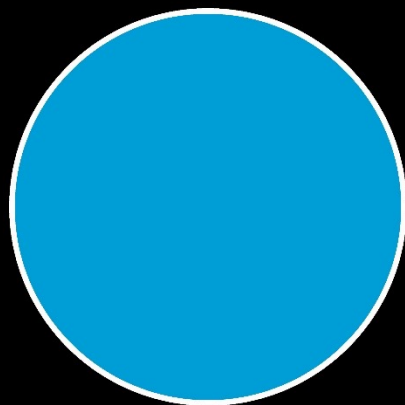
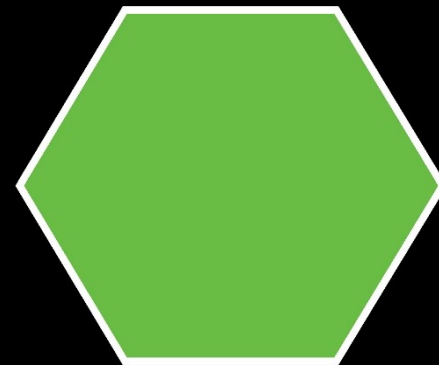
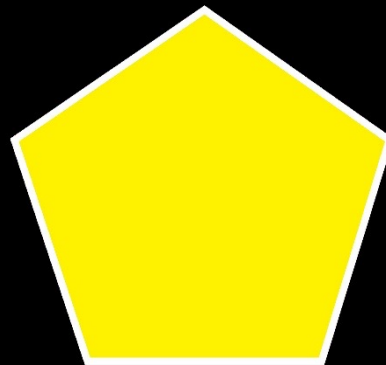


Equilateral Triangle

- 3 reflections
- 2 rotations
- “Do nothing” (Id)



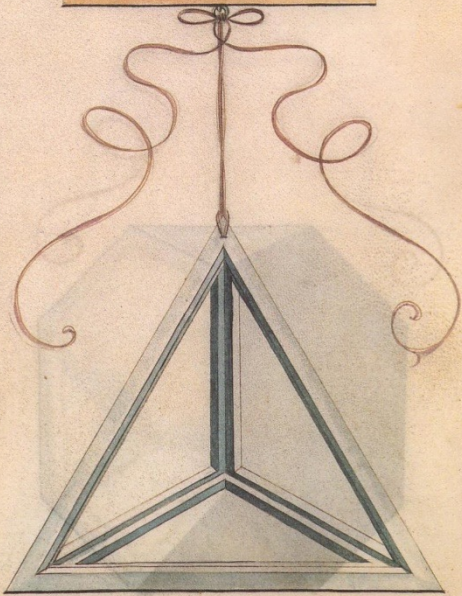
Regularity and Symmetry



Regular Polyhedra

LXXXII.

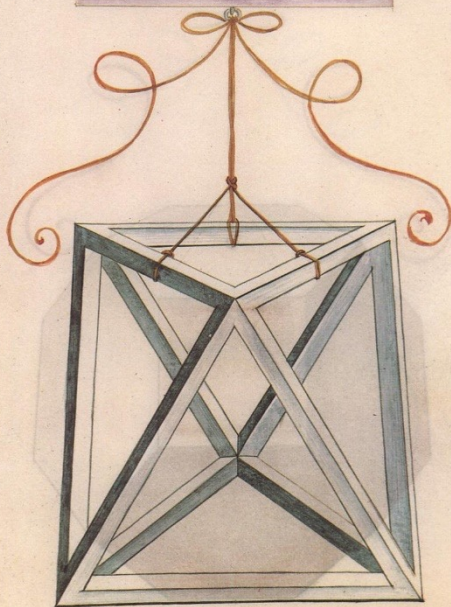
TETRACEDRON. PLANVS.
VACVVS.



Tetrahedron planus

LXXXIX

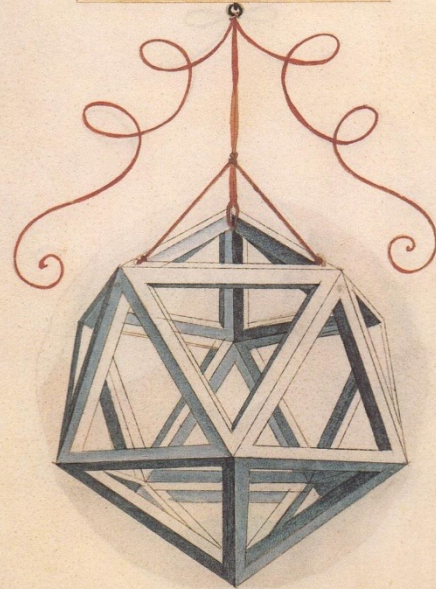
OCTOCEDRON PLANVS
VACVVS.



Octaedron planus

CII

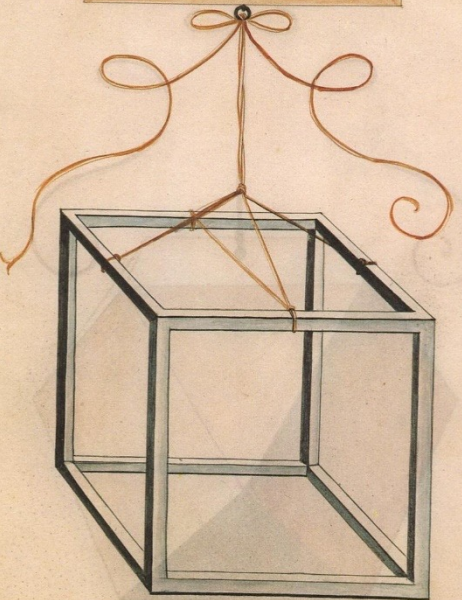
YCOCEDRON. PLANVS.
VACVVS.



Icosaedron planus

LXXXV

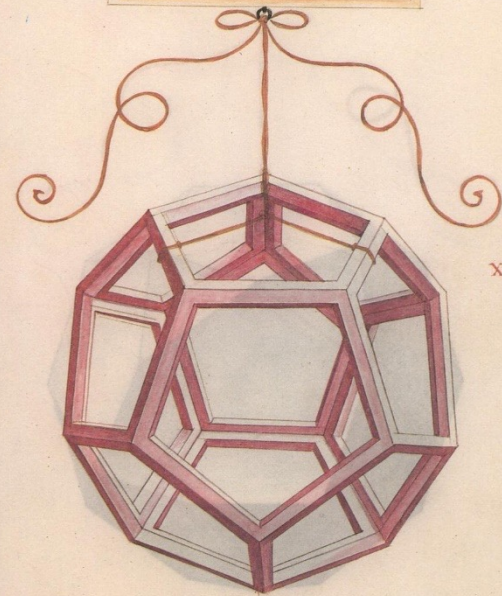
EXACEDRON PLANVS
VACVVS.



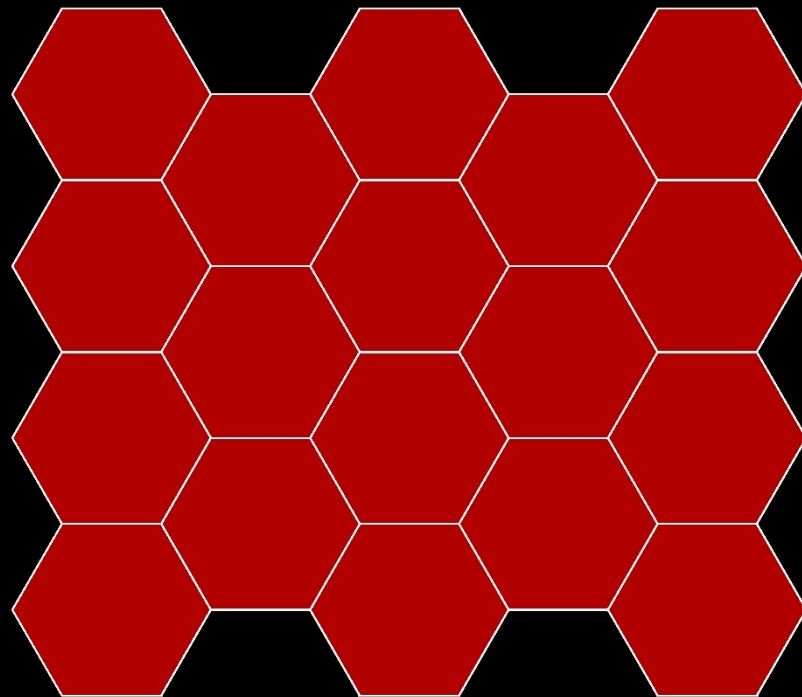
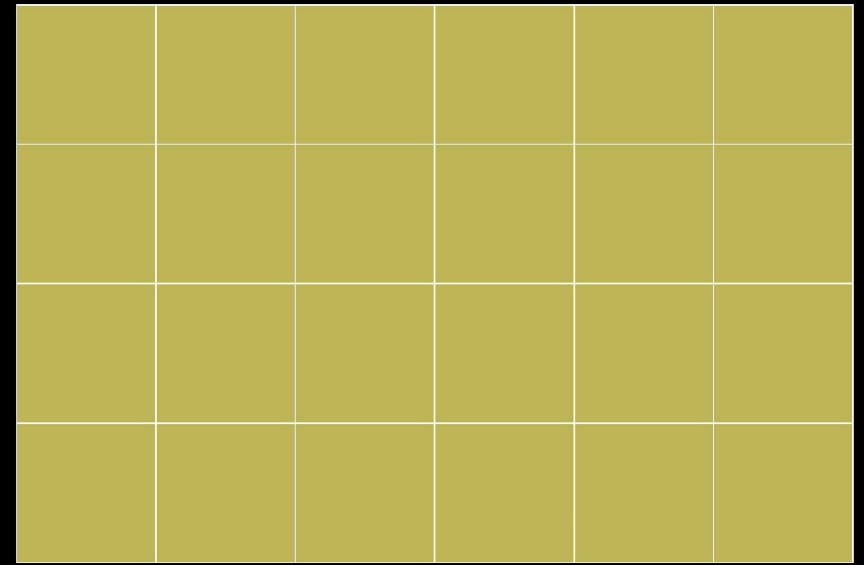
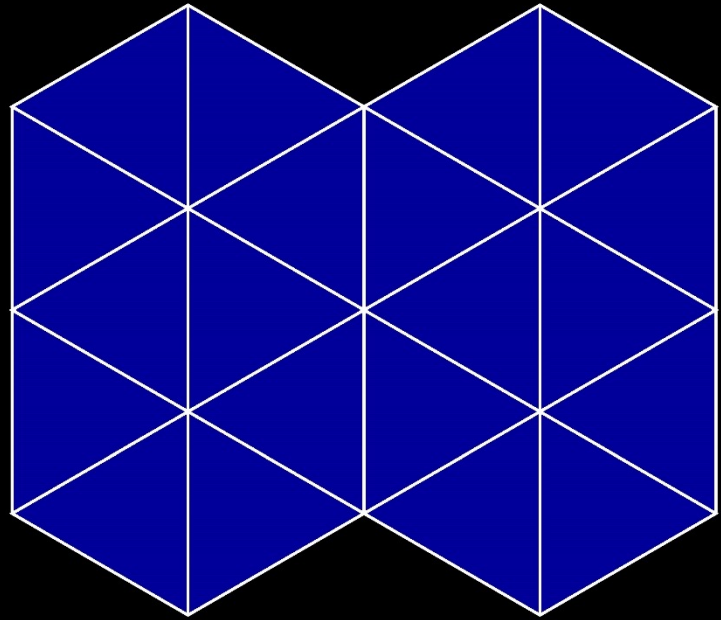
Hexaedron planus

.CV.

DVODECEDRON PLANVS
VACVVS.



Dodecaedron planus



Neolithic Platonic Solids?

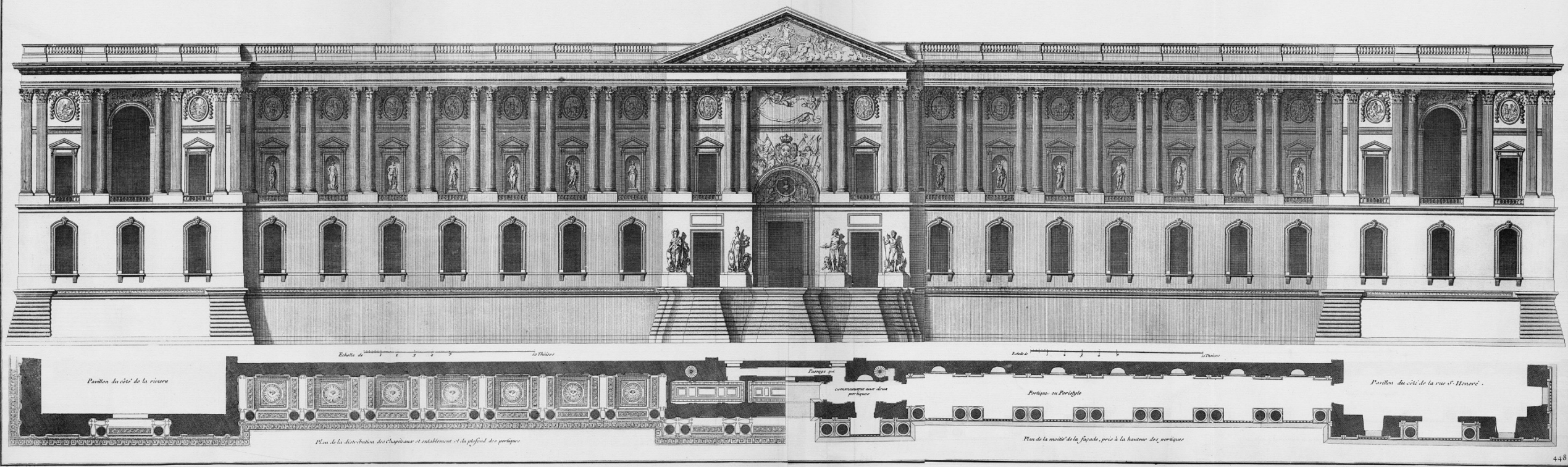






From σύμμετρία to symmetry

- σύμμετρία (*symmetria*): “syn” (together) + “metros” (measure).
- Euclid: magnitudes are *symmetros* if they have a “common measure”
- Pliny: non habet Latinum nomen *symmetria*
 - Commensuratio/commensurable
- Aristotle (*Metaphysica*): “the main species of beauty are orderly arrangement (*taxis*), proportion (*symmetria*), and limitation (*horismenon*), which are revealed in particular by mathematics”.
- Vitruvius (*De Architectura*): *symmetria* is “the appropriate harmony arising out of the details of the work itself; the correspondence of each given detail to the form of the design as a whole”.



Claude Perrault (1673): “Symmetry does not signify in French what Vitruvius understands by Proportion [symmetria]. Symmetry, in French, signifies the relation, for example, that windows have one to another, when they are all of an equal height and equal breadth; and that their number and distances are equal to the right and the left; so that if the distances be unequal of one side, the like inequality is to be found in the other.”



“Two equal solid angles which are formed by the same plane angles but in the inverse order will be called ‘*angles equal by symmetry*’, or simply ‘*symmetrical angles*’.”

Adrien-Marie Legendre,
Éléments de Géométrie (1794)

The composition of any two symmetries is another symmetry. “Closure property”

- Eg for a square, rotation through 90° then another rotation through 90° ;
- reflection in vertical line then reflection in horizontal line.
- Other properties: identity and inverses.



Symmetries and Groups

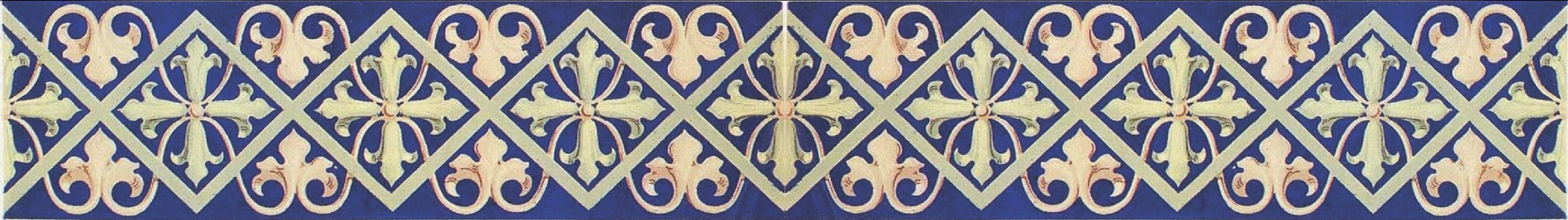
The set of symmetries of a shape is one example of a **group**.

- A group is a set with a way of combining the elements, subject to four rules.
- Closure
- Identity
- Inverses
- Associative Law

The integers with $+$ form a group.
For any a, b, c integers:

- $a + b$ is an integer.
- $a + 0 = 0 + a = a$.
- $a + (-a) = 0 = (-a) + a$.
- $(a + b) + c = a + (b + c)$

Frieze patterns



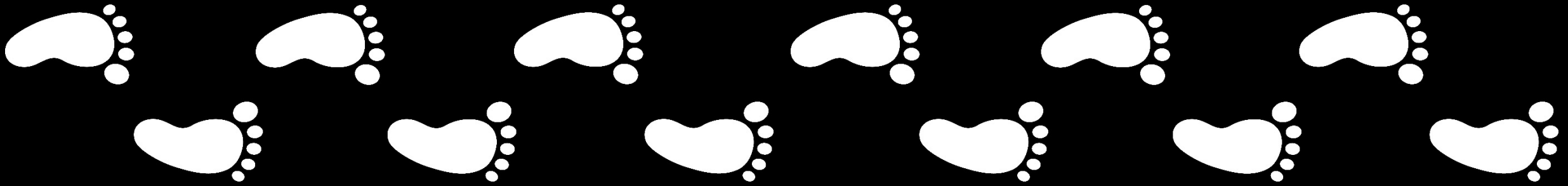
Always have translation through a whole number of steps as a symmetry.
The design to be repeated determines the rest.

Could have:

- reflection in central horizontal line
- reflections in vertical lines
- rotation 180° about point on central horizontal line



Frieze patterns

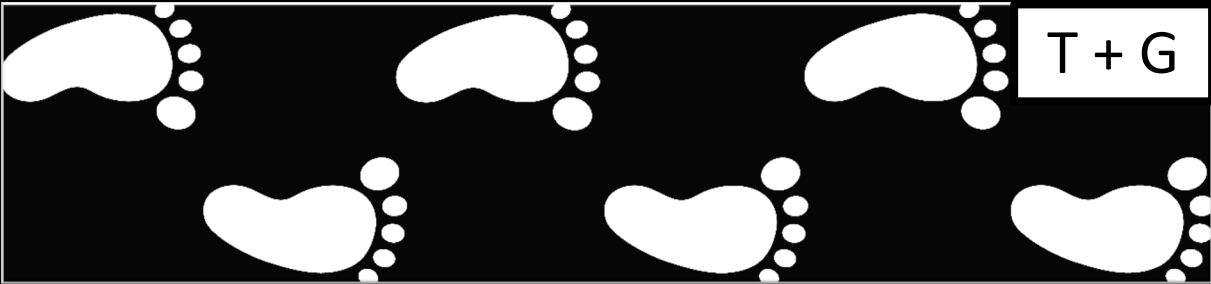


- Glide reflection: translation half a step then reflection in horizontal line.
- Not all possibilities occur independently!
- Eg if reflection in horizontal & vertical, then must have rotation 180° .
- Glide reflection + reflection in horizontal impossible.

Dance of the seven walls



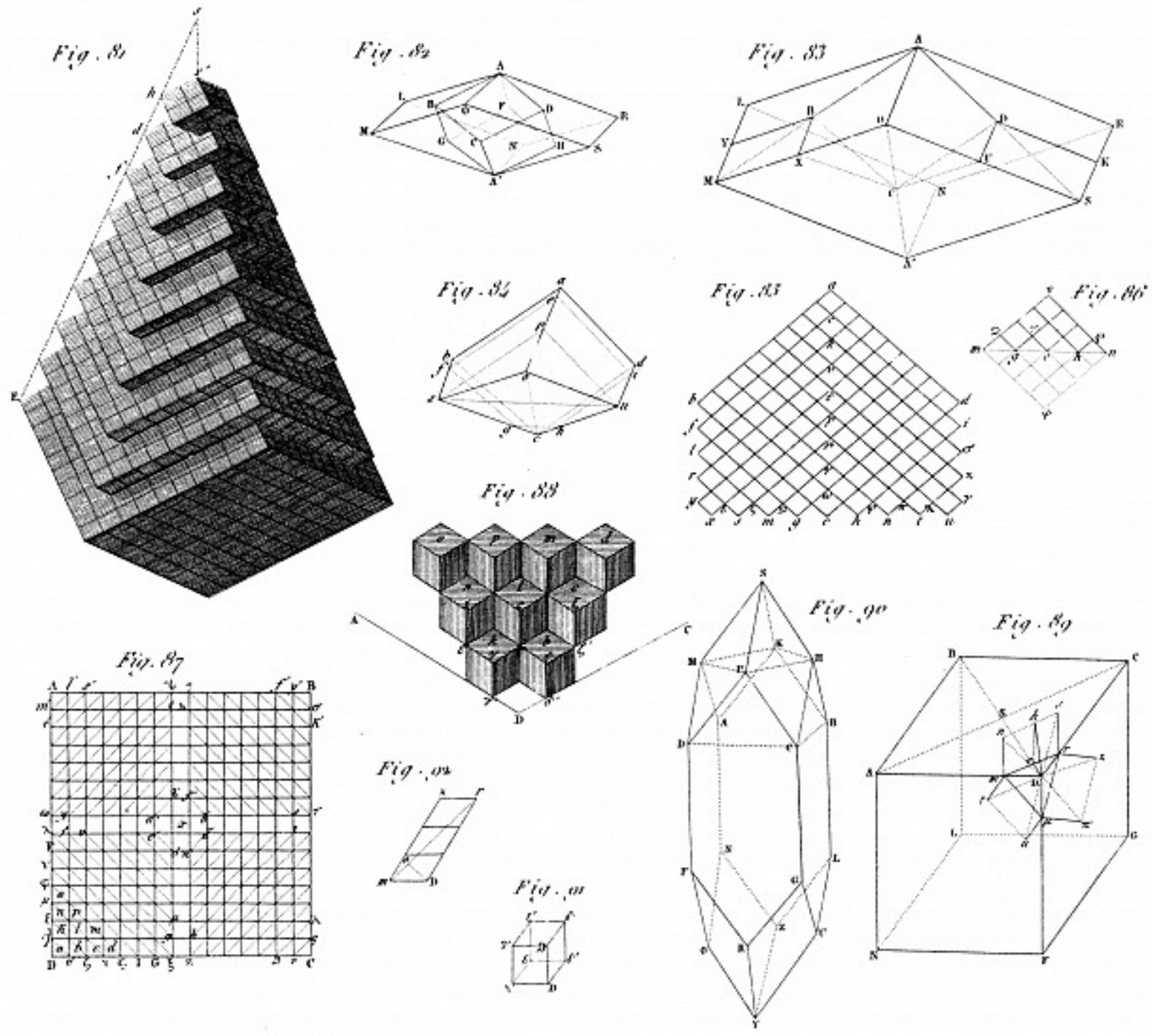
- (T) Translation [always have]
- (V) reflection in vertical line
- (H) reflection in horizontal line
- (R) rotation 180°
- (G) glide reflection

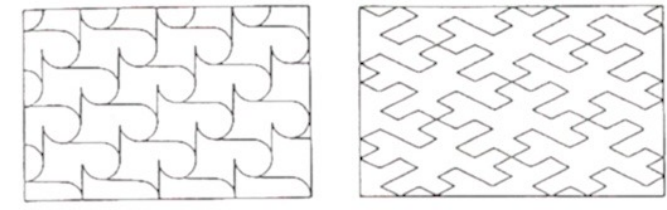


Law of Symmetry

“The way in which nature produces crystals is always that of the greatest symmetry, in that opposite and corresponding parts are always equal in number, arrangement and shape.”

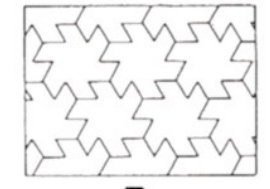
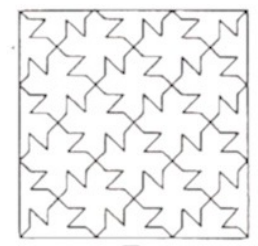
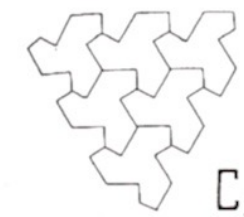
René Haüy, 1815.





C_1

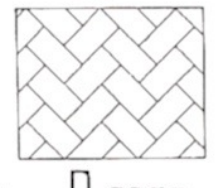
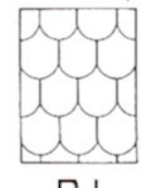
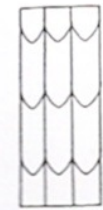
C_2



C_3

C_4

C_6



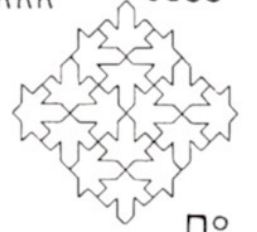
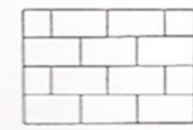
D_{1kk}

D_{1gg}

D_{1kg}

D_{2kkkk}

D_{2gggg}

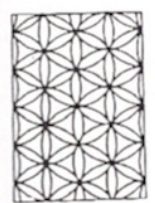
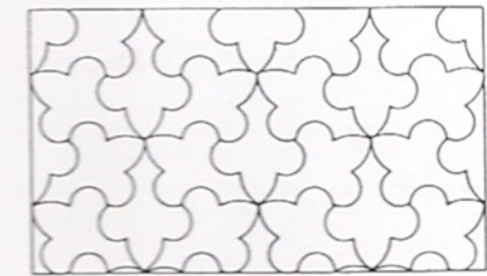


D_{2kkgg}

D_{2kgkg}

D_4^*

D_4^o



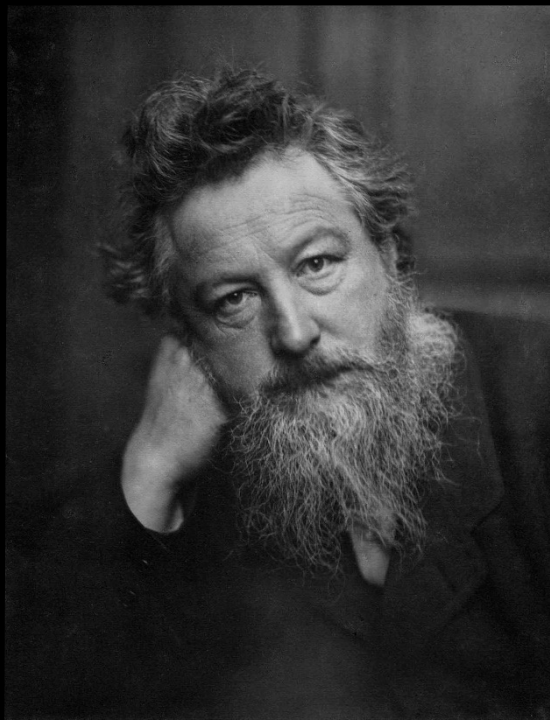
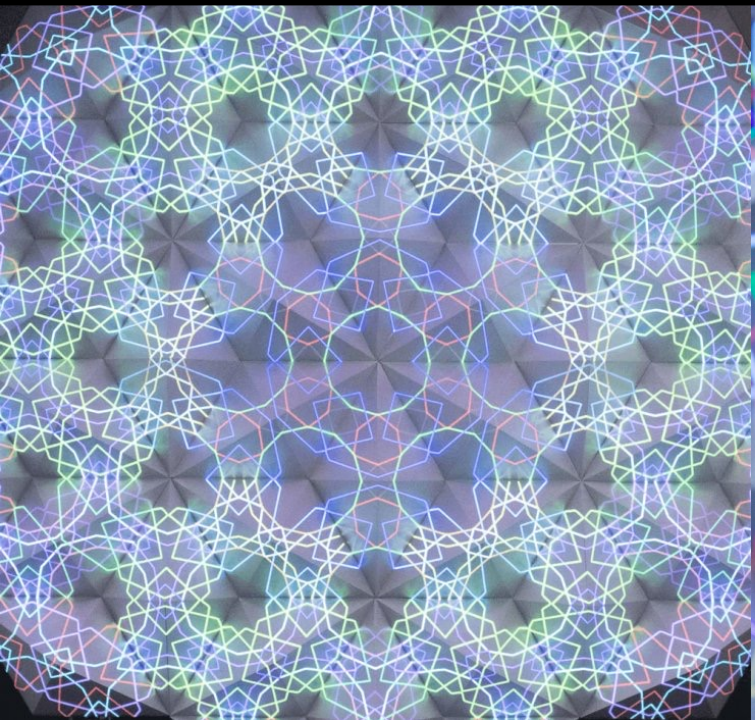
D_3^*

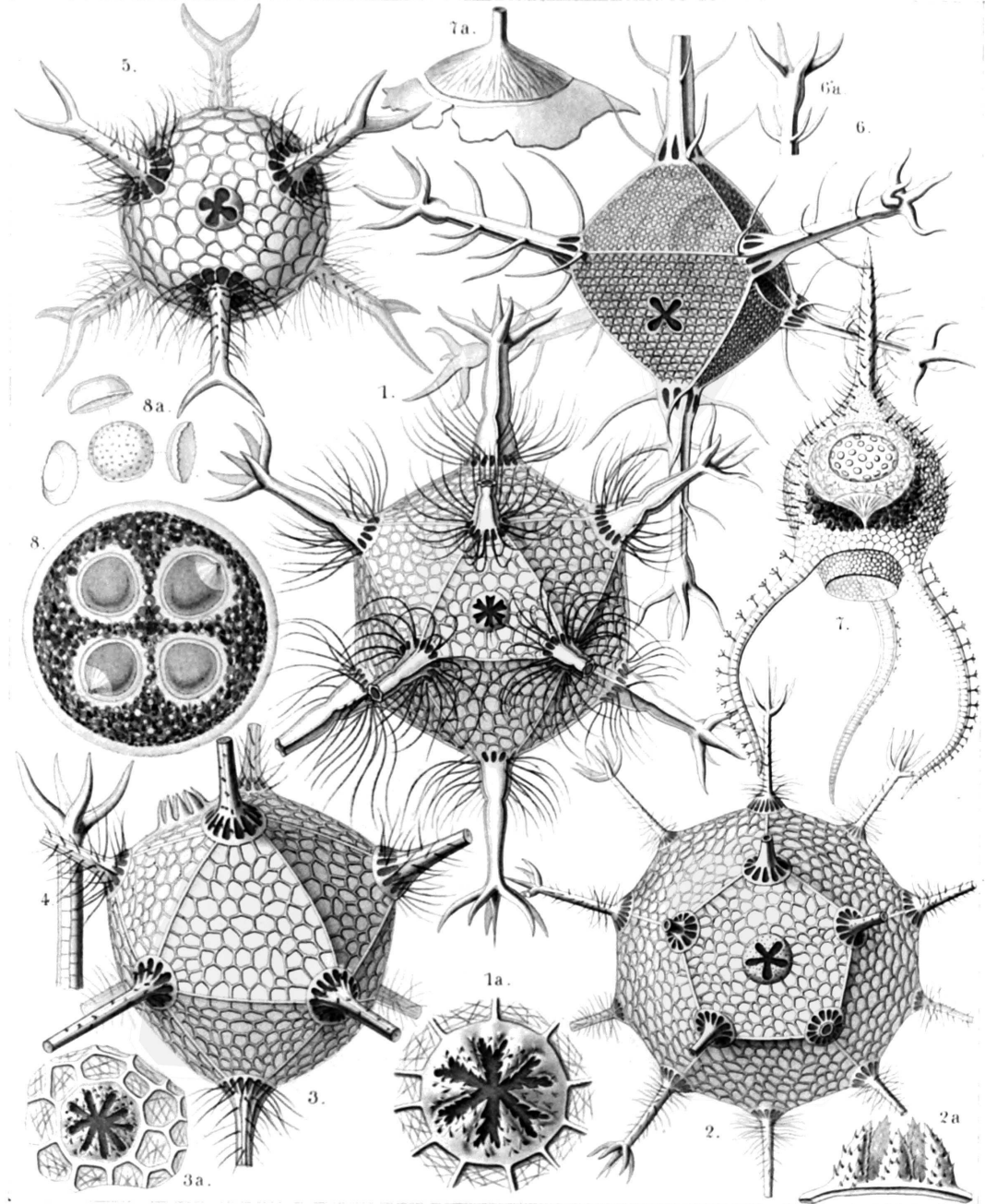
D_3^o

D_6



Fish by Chris Watson
TessellationArt.com



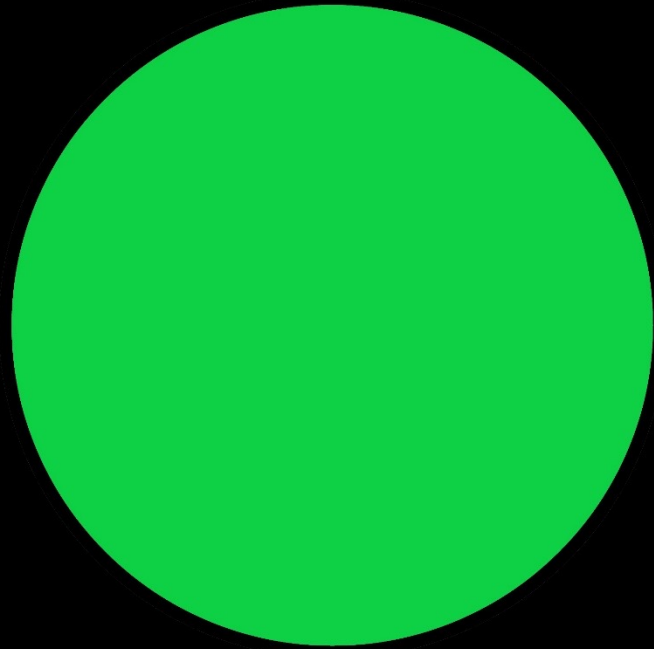


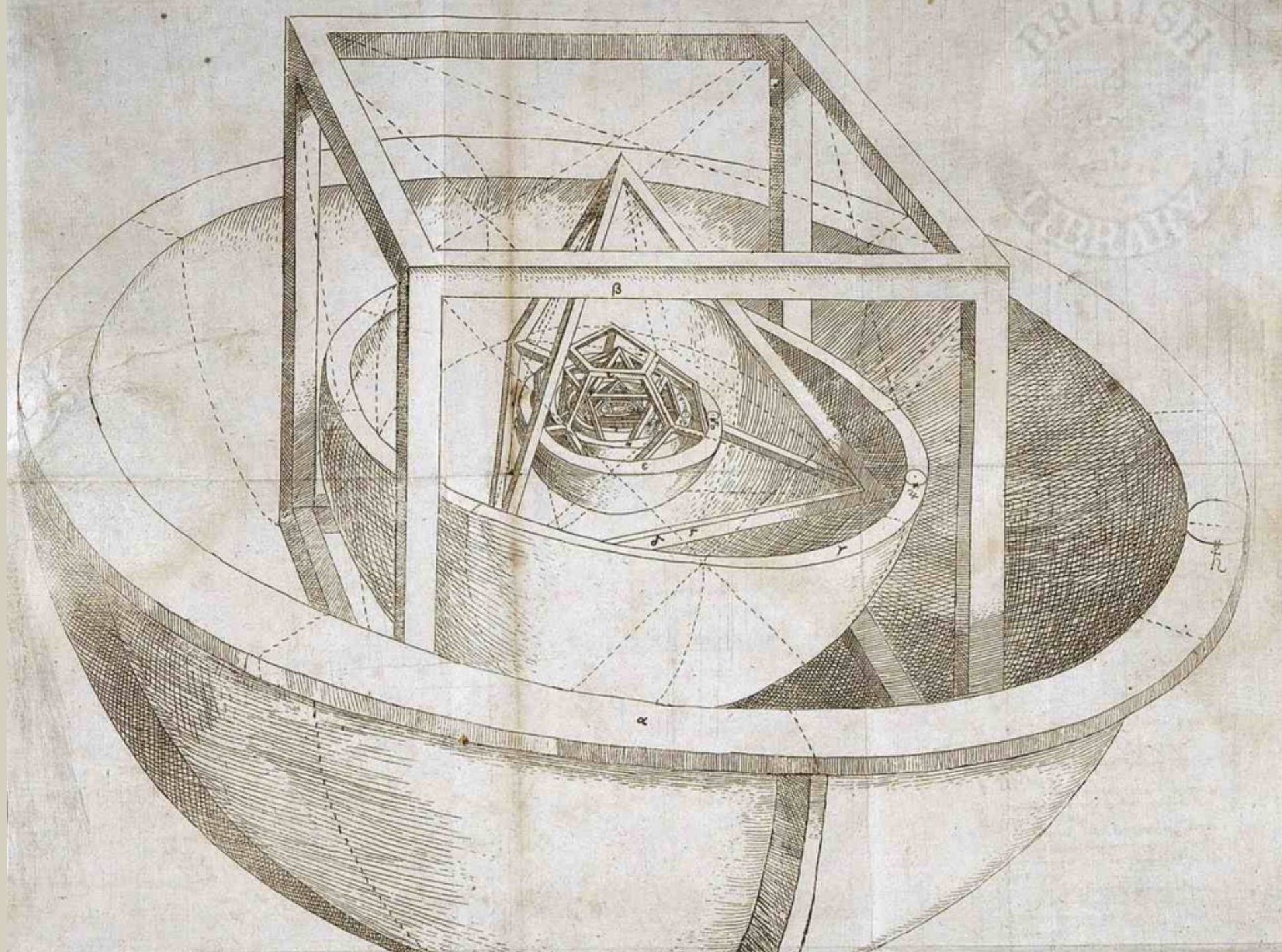
E. Haeckel, and A. Schrank Del.

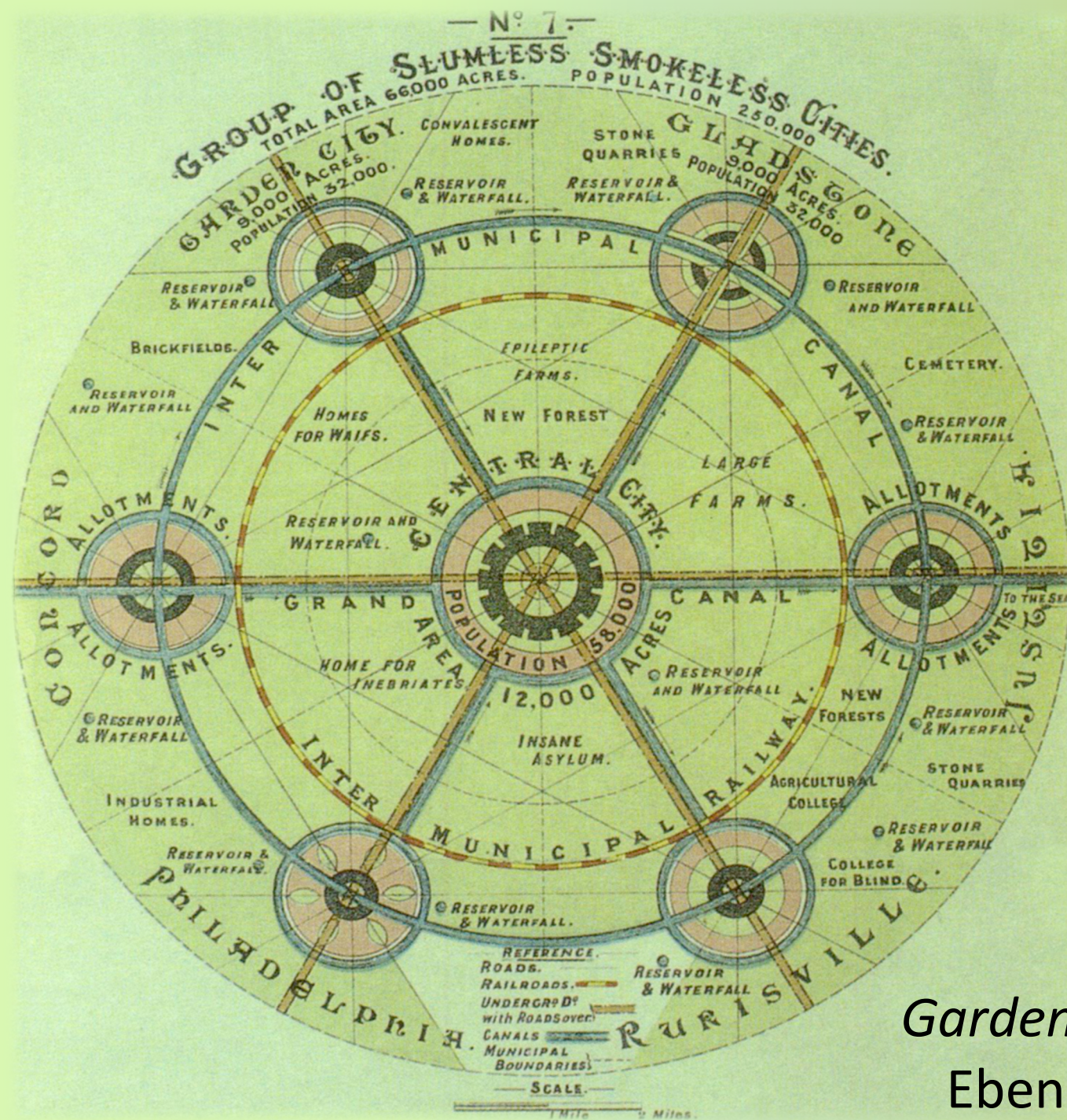
A. G. Sars, Jena, Lith. pr.

1. CIRCOGONIA . 2. CIRCORRHEGMA . 3. CIRCOSPATHIS .
4-6. CIRPOPORUS . 7. CORTINETTA . 8. CATINULUS .

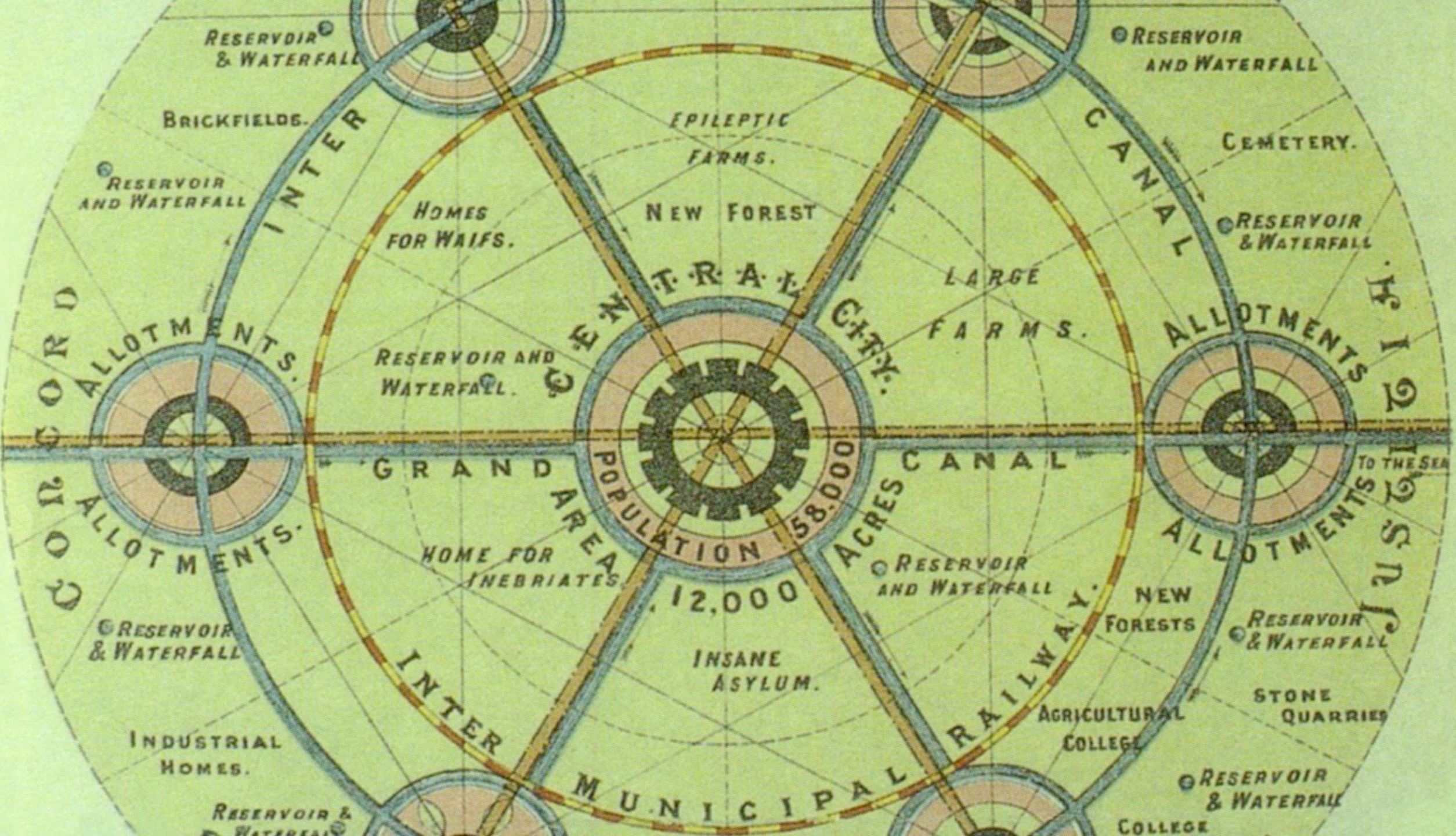








Garden Cities of Tomorrow
 Ebenezer Howard (1902)



RESERVOIR & WATERFALL

RESERVOIR AND WATERFALL

BRICKFIELDS.

EPILEPTIC FARMS.

CEMETERY.

RESERVOIR AND WATERFALL

HOMES FOR WAIFS.

NEW FOREST

RESERVOIR & WATERFALL

CORD ALLOTMENTS.

CENTRAL CITY.

LARGE FARMS.

ALLOTMENTS

RESERVOIR AND WATERFALL.

GRAND AREA.

CANAL

TO THE SEA

CORD ALLOTMENTS.

POPULATION 58,000 ACRES

ALLOTMENTS

HOME FOR INEBRIATES.

12,000

RESERVOIR AND WATERFALL

NEW FORESTS

RESERVOIR & WATERFALL

RESERVOIR & WATERFALL

INTER

INSANE ASYLUM.

MUNICIPAL RAILWAY.

AGRICULTURAL COLLEGE

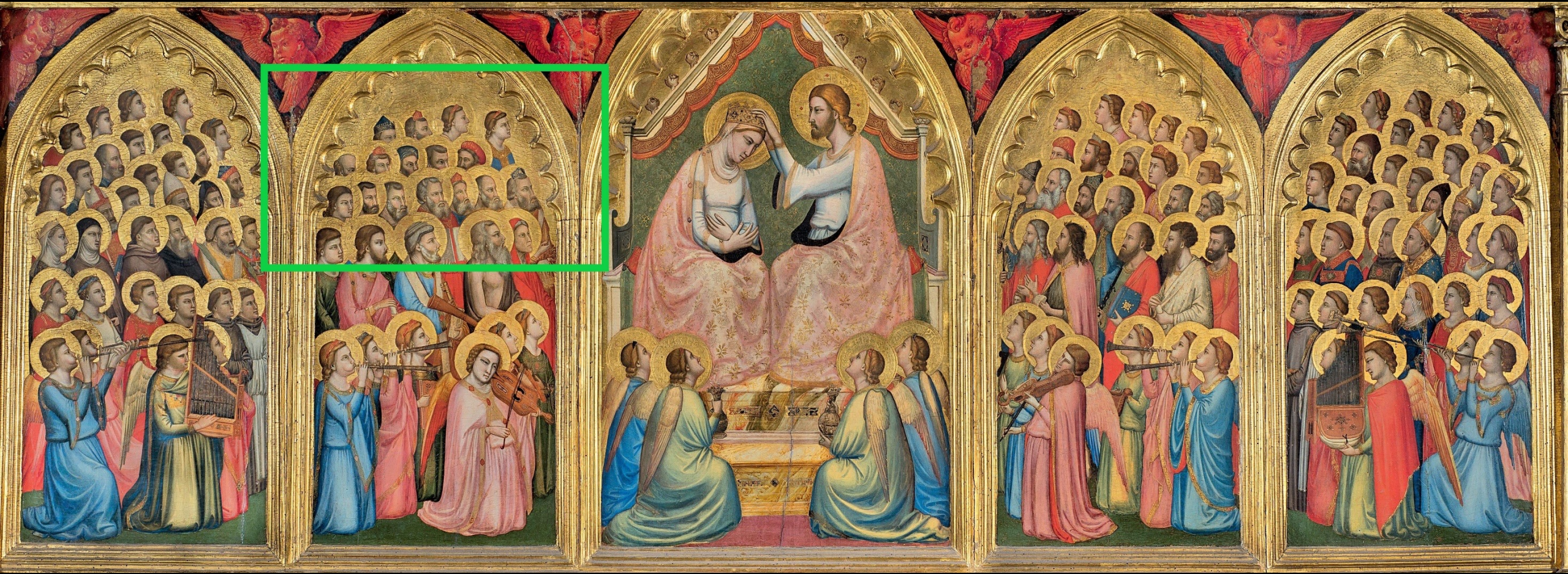
STONE QUARRIES

INDUSTRIAL HOMES.

RESERVOIR & WATERFALL

RESERVOIR & WATERFALL

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The Maths of Proportion in Art, Design and Nature

February 7th, 1pm

Gresham.ac.uk @greshamcollege

