# The Mathematics of Beauty and Symmetry

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People have always found symmetry aesthetically pleasing and examples of it are seen in the earliest art. This lecture will look at how we can understand symmetry using mathematics, and explore how the rules of symmetry can deepen our appreciation of beautiful works of art and design.

## Introduction

Throughout history, across different times and cultures, it's clear from art, design, and architecture, that humans find beauty in regularity and symmetry. We see it, for example, in the exquisite mosaic ceilings at the Lotfollah mosque in Isfahan, Iran, and the Moorish tilings at the Alhambra Palace in Spain. We see symmetry in architecture: the Taj Mahal, often said to be the most beautiful building in the world, has a left-right, or "bilateral" symmetry with a vertical mirror line. By cleverly placing a shallow lake in the gardens in front of the building, the impression is given of an additional horizontal mirror line, as the building is reflected in the still waters below. Meanwhile if we look at the famous Renaissance Villa Rotonda at Vicenza in Italy, designed by the architect Palladio, we see that it has not only a pleasing bilateral symmetry on its individual façades, but that in fact the whole building is symmetrical – the four sides of the building form a square, with a central rotunda, so that the whole building (from the outside at least) has a four-fold rotational symmetry. Looking at the original plan, we can see that the corners of the building lie on a circle, which also just touches the centres of each of the colonnades.

Human beings clearly like symmetry. But why? There are two main views on this. The first is called the "Perceptual Bias" view. Symmetrical or regular designs are literally easy on the eye, in the sense that the brain has to make less effort to process them. A chequerboard floor has an instantly comprehensible pattern that the brain can analyse very quickly. Deviations from the symmetry require special notice and can have a jarring effect. Nobody wants a tiled floor with one tile out of place (an example of which I'll show you in the lecture – sorry). The second suggestion as to why we like symmetry is called the "Evolutionary Bias" view. Our primitive animal brains, goes the theory, subconsciously size up potential mates with a "survival of the fittest" bias. Symmetrical bodies may be perceived as healthier and thus a better bet to pass on your genes. Whatever the reason, the fact that we do perceive highly symmetrical objects as beautiful has led us to look for them, and then to associate them with goodness, with virtue, even with the divine.

## Measuring Symmetry

Plato, in the *Timaeus*[1] (360 B.C.), wrote about how the universe was made, by "a method with which your scientific training will have made you familiar. Fire, Air, Earth, and Water are bodies and therefore solids [...] we must determine what are the four most beautiful figures." Which are the four most beautiful figures, then, that we must associate with the four elements? It turns out that they are the tetrahedron, the octahedron, the cube, and the icosahedron. How do we know which figure should be associated with which element? Well, when we touch fire it hurts, and the sharpest of these polyhedra is the tetrahedron – so that must be fire. By contrast, water slips through our fingers without any unpleasant sensation, so it is icosahedral. The cube is the only one of these solids that can be stacked up neatly without gaps, so clearly earth is the cube, which leaves the octahedron to be air. The illustration is the one from Book 5 of Kepler's 1619 *Harmonices Mundi*

[1] The *Timaeus* is a dialogue in which several characters, Timaeus among them, discuss the creation of the universe.

– Harmony of the World.

But, there is another solid in Plato's armoury – if you have heard of the Platonic solids you'll know this. He says that there is "a fifth figure (which is made out of 12 pentagons), the dodecahedron – this God used as a model for the twelvefold division of the Zodiac". This is the "fifth element", the quintessence. So, how did Plato come to have five "most beautiful figures" to assign special roles to? To answer that, we need to think about how to measure symmetry.

We say that a symmetry of a shape is, informally, something you can do to it that leaves it looking the same. A bit more formally, we can define it to be a distance-preserving transformation of space (such as a reflection, rotation or translation) that maps a shape to itself. This is not how Plato or Euclid would have thought about it, but we'll say a little more about that later.

As an example, an isosceles (but not equilateral) triangle has two symmetries. It can be reflected in its one line of mirror symmetry, so that every point is mapped to the point the same distance from the line on the other side. The other symmetry is easily overlooked, but the identity map, where every point is simply mapped to itself, is a symmetry of every shape. Given that a symmetry has to preserve distances, it must map edges to edges, so the fact that one edge has a different length from the other two limits the possibilities. That odd edge can only be mapped to itself. Similarly, two edges that meet at a specific angle must still meet at that same angle after the symmetry operation has been completed. That leads us to suspect that we might get more symmetry in a triangle where all the sides are equal lengths and all the angles are equal. Sure enough, when we analyse an equilateral triangle, we find that there are now six symmetries. We get three reflections, two rotations (through $120°$ and $240°$) and of course the identity map (which we can think of as a rotation through 0 degrees). Thinking about polygons in general, the same reasoning tells us that the most symmetrical $n$-gon is a regular $n$-gon, that's one with equal sides and equal angles. A square has eight symmetries: four reflections and four (if you include the identity) rotations. A regular pentagon has ten. A regular $n$-gon has $2n$ symmetries. If we take the limiting case, we end up at a circle – every rotation about the centre, and every reflection in a line through the centre, is a symmetry.

Moving up a dimension, let's think about polyhedra. These are three-dimensional shapes made with straight edges, whose faces are polygons. Again, for maximum symmetry we would want all edges to be the same length, and all angles to be equal. That is, we would like the faces to be regular polygons, and the same number of faces to meet at each vertex. What are the possibilities for these "regular polyhedra"? If the faces are equilateral triangles, how many could meet at each vertex? Two is not enough, because they collapse on each other and we cannot get a three-dimensional shape. Three, four, and five are candidates. But six equilateral triangles will give a total angle of $6 \times 60° = 360°$, which would just be completely flat. Again, we wouldn't obtain a three-dimensional shape. Testing out the three candidates does indeed result in three regular polyhedra: the tetrahedron, the octahedron, and the icosahedron. With squares, we are more constrained. Two is again not enough. The internal angle of a square is $90°$, so four squares round a point makes $360°$, which would be flat and not give a three-dimensional shape. So our only hope is three squares round a point. This works: we get a cube. The internal angle of a regular pentagon is $108°$. So three round a point is possible. It gives the dodecahedron. Four regular pentagons wouldn't even be flat: we'd actually have them overlapping. Anything higher than pentagons doesn't work because two is not enough, and three is too many to fit round a point – you end up with an angle of at least $360°$. And that's why we have exactly five regular polyhedra – known as the five Platonic solids.

For me, this highlights one of the precious things about mathematics. Our scientific understanding of the world is completely different now from what it was two thousand years ago, and we might smile at the shoehorning of the five solids into a scientific theory of the universe. (It has to be said, though, that modern humans are hardly immune from the temptation to coerce data to fit preconceived ideas.) However, the fact that there are exactly five regular polyhedra is as true today as it was then. Mathematical proof is eternal. Incidentally, those limiting cases where we end up with exactly $360°$ from a whole number of regular polygons are not meaningless – they are precisely the possibilities for what's called regular tilings of the plane: tessellations where each tile is the same regular polygon. There are three of these: the triangular, square, and hexagonal.

# The history of symmetry

Periodically the claim will resurface that hundreds of years before Plato, Neolithic tribes knew about the Platonic solids. The evidence for this is the hundreds of carved stone balls found at various sites in Scotland, dating back to around 2000BC. There is a compelling image of five of these stone balls with the edges emphasized by strips of white paper, looking very much like the five platonic solids. Unfortunately there does seem to be quite a bit of wishful thinking here. The stone balls all have bumps in them, and these bumps can be interpreted as either faces or vertices. So we can interpret something with six or eight bumps as a cube (six faces, eight vertices) or an octahedron (six vertices, eight faces). Tetrahedra have four faces and four vertices. For icosahedra and dodecahedra these numbers are twelve and twenty. Of the hundreds of stone balls found, the number of bumps ranges from 3 to over 100 (I've seen both 135 and 160 stated as the maximum number). The ones that happen to have four, six, eight, twelve, or twenty bumps don't seem to have any particular significance or prevalence. Around half the balls have six bumps. Just eight have twelve bumps, and only two have twenty bumps, with lots of other numbers represented that have no apparent significance. This is not very compelling evidence for Neolithic Platonic solids. These are very interesting archeological finds already, without imposing on them symbolism that almost certainly isn't there. If you want Platonic solids in archeology, you can have them, but you have to wait a little longer, probably until the metal dodecahedra found in Roman settlements. Their purpose, as with the Neolithic balls, is unclear. Theories include them being knitting aids, gauges for pipes (the holes all have different diameters), or gaming pieces. We may never know. But there's no doubt that they are dodecahedra.

I have mentioned that the modern definition of symmetry would not be familiar to Euclid and Plato. Ancient Greek had the words σύμμετροζ (*symmetros*) and σύμμετρία (*symmetria*). This is made up of the root words "syn" (together) + "metros" (measure). This word appears in Euclid's Elements (Book 10, Def 1), but not in a way that has anything to do with our idea of "symmetry". Two magnitudes are *symmetros* if they have a "common measure" (that is, they are both whole number multiples of a given amount). Platonic solids are constructed in Book 13 of Euclid, but they are not described as being symmetrical. Rather, they are "the solids made of equilateral figures equal to one another". When Greek texts were being translated into Latin, a decision had to be made about how to translate *symmetros*, because, according to Pliny in his Natural History (77AD), "non habet Latinum nomen *symmetria*": there is no Latin word for *symmetria*. Translations into Latin, and much later into English, of this rather technical geometric meaning of the word *symmetria*, use "commensuratio" and "commensurable" (having a common measure). However, this same word *symmetria* was also used in Greek in a broader aesthetic sense of magnitudes being connected or related: being in proportion. Aristotle (384-322BC) in his *Metaphysica* tells us that "the main species of beauty are orderly arrangement (*taxis*), proportion (*symmetria*), and limitation (*horismenon*), which are revealed in particular by mathematics". This meaning transferred to Latin, along with the word. Vitruvius in *De Architectura* says that *symmetria* is "the appropriate harmony arising out of the details of the work itself; the correspondence of each given detail to the form of the design as a whole".

By the late 17$^{th}$ century we start to get clear evidence that the word "symmetry" had changed its meaning. The architect Claude Perrault (1613-1688), probably most famous now for designing the Louvre colonnade, translated the works of Vitruvius into French, along with explanatory notes. "Symmetry", he wrote, "does not signify in French what Vitruvius understands by Proportion [symmetria]. Symmetry, in French, signifies the relation, for example, that windows have one to another, when they are all of an equal height and equal breadth; and that their number and distances are equal to the right and the left; so that if the distances be unequal of one side, the like inequality is to be found in the other. [Whereas] the word Simmetria signifies in Latin and Greek relation only. As for example, as the relation that windows of eight foot high, have with other windows of six foot, when the one are four foot broad, and the other three. For our symmetry is properly the equality and the parity which is found between opposite parts, that is, if one makes, for example, an eye higher or larger than the other, or columns closer together to the right than to the left, or [their] number or size is not the same [to the right and to the left], one says that it is a defect of symmetry according to us: rather than, if a capital is larger or a cornice protrudes more than the rules of order for a column allow, this is a defect of symmetry according to the ancients." This Latin and Greek "symmetria" is nowadays translated as "proportion". That is the subject of my third Gresham lecture of this academic year, in February 2022. What we are discussing today is Perrault's "French" definition of symmetry, where different parts are alike or equal, such as the left-right bilateral symmetry that is common in classical architecture.

# Groups

Symmetry, in its new “French” meaning, initially meant bilateral symmetry. Architectural manuals of the time would say, for example, that if you had a window or door on the left of a façade, you should have a corresponding one (fake if necessary) on the right. Thinking of symmetry like this makes it static: a property of an object rather than something you do. The definition of “a symmetry” as we have given it was not possible with this mindset. Mathematically, the crucial first step to our modern understanding was in a definition by the French mathematician Adrien-Marie Legendre, in his book *Éléments de Géometrie*, in 1794. “Two equal solid angles which are formed by the same plane angles but in the inverse order will be called ‘angles equal by symmetry’, or simply ‘symmetrical angles’.” What this does, for the first time, is to turn symmetry not into a property but a relation: “by symmetry”. The symmetry is doing something rather than being something – it’s turning into a verb! It’s rather like what had happened a century or two earlier with the study of curves in geometry. Mathematicians like Descartes had started to think of curves not simply as static objects, but as representing an active relationship between variables, where we plot $y$ as a function of $x$ or maybe of time. This laid the path for the development of the calculus later on. So what can we do with the idea of symmetry as being active: something you do to an object that leaves it looking the same?

When you play around with symmetries, you notice that combining them results in more symmetries. With a square, for instance, the composition of any two of the reflection symmetries results in a rotational symmetry. If you combine a rotation with a reflection, you get another reflection. The same is true for any shape. If $f$ and $g$ are symmetries then $f$ followed by $g$ is also a symmetry. There are some other properties too. We already spotted that every shape has the “do-nothing” identity map as a symmetry. Every symmetry is also reversible – it has an “inverse” which is also a symmetry. For instance, rotation through 90 degrees anticlockwise can be undone by rotating through 90 degrees clockwise.

The set of symmetries of a shape is one example of the mathematical structure known as a group. In the 19th century, many different areas of mathematics started noticing and then using the same kind of structure, one in which elements of a set could be combined in some way and the outcome was still in the set. The first person to actually use the word “group” was Évariste Galois, in about 1830. He was studying the problem of whether there are formulae like the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for higher order polynomials like those involving $x^3$ (cubics), $x^4$ (quartics), or $x^5$ (quintics). What Galois did was to think about the roots of polynomials in terms of ways they could be shuffled around, or permuted, to still end up with roots. Just like shuffling a pack of cards, if you shuffle in one way and then shuffle in another, the net effect is another, perhaps more complicated, shuffle. So, there is this property of “closure”, and that was how Galois defined a group. This kind of set-with-an-operation was appearing elsewhere in mathematics too. You find it in numbers, in matrices, in functions, everywhere. We first see groups appearing in geometry around the 1850s, with the work of people like Möbius and Felix Klein (of Möbius strip and Klein bottle fame). They and others realised that the key to understanding different kinds of geometry was first to work out what are the things that matter to you (so in Euclidean geometry that would be distances between points), and then look at the groups of transformations that preserve those things. Our symmetries are distance-preserving maps – they don’t stretch space. But if you are studying topology or non-Euclidean geometry, you might want to focus on other characteristics. We tend to simply state the “four axioms of a group” as if recorded on stone tablets in antiquity. But actually, it took decades for the definition to be refined into its current form. Just for the record, here are the axioms, with the example of the set of integers (whole numbers), where the operation (the way of combining the elements) is addition, as an example.

A group is a set $G$ with an operation $*$ defined on it, so that the following four rules apply

- For any elements $a$ and $b$ of $G$, $a * b$ is in $G$ (closure).
- There is an element $e$ such that $a * e = e * a = a$ for all $a$ in $G$. (identity)
- For every $a$ in $G$ there is an element $a'$ in $G$ such that $a' * a = e = a * a'$. (inverses)
- For every $a, b, c$ in $G$, we have $(a * b) * c = a * (b * c)$. (associative)

All these axioms are true for the integers, with addition.

- Given two integers $a$ and $b$, their sum $a + b$ is also an integer.
- Zero is the identity element because $a + 0 = a = 0 + a$ for any integer $a$.
- The additive inverse of a is $-a$. That is, $a + (-a) = (-a) + a = 0$.
- Finally, the associative law holds for addition; it’s always true that $(a + b) + c = a + (b + c)$.

Group theory is incredibly powerful. Anything that we can prove starting with these four initial rules will give us information about any context where there are groups: numbers, geometry, symmetries, functions, and so on.

## Friezes and tilings

Now we know what symmetry groups are, let's see how they can be used to understand and classify a particular kind of design seen in decorative art: the frieze. Mathematically speaking, we think of a frieze as a repeating pattern along a strip that could be continued indefinitely. So one symmetry that's always present is translation through a whole number of units (where a unit is the smallest distance after which the pattern repeats). What other symmetries there are depends on the design that's being repeated. We can actually classify the different possible patterns according to their symmetry groups. We begin by deciding the list of potential symmetries (apart from translations and the identity map). The central horizontal line must be mapped to itself by any symmetry. So the only rotations possible are rotations of $180°$ about points on that line. The only reflections possible are in vertical lines or in the central horizontal line. There's one other kind of symmetry that may be a bit less familiar: glide reflections. A glide reflection is a half-unit translation followed by reflection in the central horizontal line. Footprints have this kind of symmetry.

By the way, it can be shown – we probably won't have time for this in the lecture – that if there are rotations, then we can assume they are about points at the midpoints and endpoints of the pattern. The reason is that the product of two rotations of $180°$ about different points is a translation in the direction of the line joining them through twice the distance between them. So if there were two rotations closer than that, they would imply the existence of a translation symmetry through less than one unit. But we defined a unit to be the smallest repeating component. The product of two reflections in vertical lines is a horizontal translation through twice the distance between them. So the distance between any two vertical reflection lines can only be a half integer multiple of the unit distance.

The upshot of all this is that there are exactly four possible non-identity non-translation symmetries. They are:

(R) $180°$ rotations about the midpoints and endpoints of the design.

(V) Reflections in vertical lines through the midpoints and endpoints of the design.

(H) Reflection in the central horizontal line.

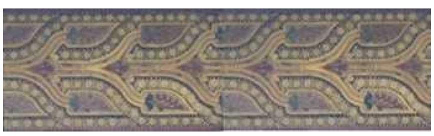
(G) Glide reflection

However, not all of these can happen independently, and this is because of the closure property of groups. I'll just give one example of this, but there are several other implications. If we have reflection in a vertical line and in the central horizontal line, then their composition must also be a symmetry – it is a $180°$ rotation. So V and H together implies R.

It takes a bit of work, but reasoning like this allows us to narrow down the possibilities to exactly seven frieze patterns. Here are real-life examples of the seven patterns, with the letters by them indicating the elements of the symmetry group. T represents translations, which every frieze group has, and included in this is translation through 0 units, which is the identity map.

T only

T + G



T + H

T + R + G

T + V

T + H + V + R

T + R

Friezes are everywhere, in decoration on the side of buildings, in ornamental borders of tiled floors, walls and ceilings: I encourage you to become a frieze pattern spotter in your everyday life.

## The Utility of symmetry

At the same time as groups were appearing in mathematics, symmetry was becoming important scientifically. It was suddenly everywhere in the 19th century, and I'll just give a couple of examples. The French mineralogist René Haüy, often called the father of crystallography, propounded his Law of Symmetry in 1815. "The way in which nature produces crystals is always that of the greatest symmetry, in that opposite and corresponding parts are always equal in number, arrangement and shape." Crystals are three-dimensional, but they are built up of two-dimensional layers, and so an important step to understanding the possible symmetries of crystals is to understand the symmetries of repeating designs in the plane. It's a bit more effort, but it's possible to classify all possible symmetry groups of such designs, and it turns out there are exactly seventeen of them. The mathematical analysis of possible crystal shapes has a link to the decorative arts. We've talked about friezes, which cover one strip of space. But if you want to cover the whole plane, then you need tiling, or tessellation, or wallpaper. (These 17 groups are often called the 17 wallpaper groups.) When the artist M.C. Escher was experimenting with tilings following a visit to the Alhambra Palace, his older brother Berndt, a Professor of Geology, recognized that the patterns were like the ones crystallographers use to categorise different crystal structures. Berndt sent his brother papers about crystal structures, including ones by George Pólya, who had independently discovered the 17 patterns in 1924 (unaware that they had in fact first been classified by the Russian mathematician and crystallographer Evgraf Fedorov in 1891). It would take too long to list all these patterns, but one thing to note is that each has one of the three regular tilings underlying it, as we can make a lattice from the central points of each design. Thus, any wallpaper pattern or tiling will have an underlying triangular, square, or hexagonal structure. By the way, since crystal structures are actually three dimensional, to understand the full set of possibilities for those we need to know not just the symmetry possibilities for the two-dimensional layers, but for the whole three-dimensional structure. These are known as "space groups". This is an even bigger undertaking, but Fedorov managed to do it: there are 230 of them. Many artists since Escher have experimented with tessellation art. I'll show an example by digital artist Chris Watson, a lovely design with fish swimming around in a regular hexagonal tiling. Artists and designers continue to work with plane-filling designs, and we see them all around us. Think Orla Keily, Enid Marx (who designed many of the fabrics used on the London Underground), and of course William Morris with his wallpapers. Artists from the Islamic tradition, such as Zarah Hussain, have brought classical patterns up to date by incorporating projections of lights that change according to a carefully designed computer algorithm.

In the natural world, large animals usually have bilateral symmetry. Because of gravity our bodies are almost guaranteed to lack vertical symmetry. We move and see, so we have a front and back. This leaves less scope for symmetry, and bilateral is about the best we can hope for. But the fewer constraints there are, the more symmetry we tend to find. As microscopes improved, it became possible to study even the tiniest creatures. Ernst Haeckel (1834 - 1919) discovered and drew thousands of microscopic organisms with high degrees of symmetry, publishing books such as *Morphological Symmetries in the Animal Kingdom*. Haeckel was a German naturalist and philosopher who discovered thousands of new species and coined many terms in biology, including ecology, phylum, phylogeny and stem cell. His diagrams of radiolara (single-celled organisms around a tenth of a millimetre in diameter) show highly symmetrical, platonic solid-like forms. In biology then, the adding of constraints results in the breaking of symmetry. By the same token, in trying to find optimal solutions to any sort of physical problem, we can appeal to symmetry. Consider the legend of Dido[2], as recounted in Virgil's *Aeneid*. Dido's father was king of Tyre, and when he died her brother

[2] The picture shown in the lecture is *Dido and Aeneas*, by Pierre-Narcisse Guérin, c1815.
Available at the Web Gallery of Art (www.wga.hu)

Pygmalion killed Dido's husband Sychaeus. The ghost of Sychaeus appeared to Dido in a dream, told her where his treasure was hidden, and that Pygmalion had killed him. Dido fled with the loot and ended up in Carthage (in what's now Tunisia, in North Africa). The local people agreed to her offer to exchange some of that fortune for whatever amount of land she could contain within the hide of an ox. Of course clever Dido cut the hide into strips and attached them at their ends to obtain effectively a long rope. The question then becomes: given a fixed perimeter with which to enclose an area, how do we maximise that area? How, in other words, can Dido get the most land? There might potentially be lots of solutions but suppose you have a candidate solution shape. Pick any point on the perimeter and draw a line $L$ to the point halfway round the perimeter on the other side of the shape. Whichever half contains the most area, we can replicate on the other half, by just reflecting the design in $L$. So there is a solution at least as good that has $L$ as a line of symmetry. But we could do this anywhere on the shape. Therefore, one optimal solution has the property that it is maximally symmetrical – it is a circle. A formal mathematical proof that this is the unique best solution takes a bit of work, but the underlying intuition is sound. Well, the upshot is that Dido became Queen of Carthage. Solving problems by exploiting symmetry is absolutely everywhere in mathematics, and it's one of the reasons why symmetry is useful and studied. And, of course, nature does this too – crystals, unaffected by the constraints of biology, have highly symmetric structures because the best solution in one direction is the best solution in all directions.

## The symbolism of symmetry

I want to finish today with a glance at the symbolism of symmetry. Symmetry, to Plato, is about form, beauty and regularity – the icosahedron, say, is optimally regular, hence "good". Centuries later, Kepler's mysticism about number and geometry being at the heart of the creator's designs meant that he was absolutely certain there had to be a reason for every number in the solar system. He was so familiar with the Platonic solids that he recognised ratios of average diameters of planetary orbits as being, to a good approximation, the same as ratios of the diameters of inscribed and circumscribed spheres of Platonic solids, as long as you put them in the correct order. There's a beautiful illustration of this in his *Mysterium Cosmographicum.* This is a continuation of the Platonic idea that symmetry is good, the creator is good, therefore the creator will naturally use symmetrical things in his designs.

Symmetry is also associated with fairness and equality. Ebenezer Howard was an English town planner, and he founded the garden city movement which envisioned towns and cities designed for people to flourish and live in harmony with nature. The book "Garden Cities of Tomorrow" (1902) included a strikingly symmetrical design for a "group of slumless, smokeless cities" with homes in smaller towns around a central city, near the countryside, in a hexagonal design. Centuries earlier, town planners in Renaissance Italy were also thinking about the ideal, utopian town, where everyone would be equal and have fair access to the town's resources. Of course, such a town should be beautiful too, and thus a symmetrical solution must be the right approach. Such a settlement, called Palmanova, was designed in the late 16th century. Unlike Howard's smokeless city, this design was actually constructed, and it's still there today.

Once we know the rules of symmetry, we can choose, on occasion, to break them. This act can have its own symbolism. In the Islamic faith there is a traditional belief that only Allah can make something perfect, and to do so yourself is thus an insult to Allah. So, many handmade Persian rugs will contain a small deliberate mistake somewhere, a tiny break in the symmetry of the intricate design. Another, more earthly, example of symmetry-breaking is in the Baroncelli Polyptych, and we'll finish the lecture with this little puzzle. This is a series of five panels by Giotto, the only of his panel paintings that is still in the place it was designed for, the Baroncelli chapel in the church of Santa Croce in Florence. The central panel depicts the Virgin Mary being crowned Queen of Heaven, with an array of saints and angels symmetrically placed on either side. On each side there are 51 saints and ten angels. The use of symmetry gives a harmony to the work and draws the eye to the centre. All 61 figures on the right are looking towards the central panel. All 61 figures on the left are looking towards the central panel. Or are they? If we look closer, in fact one of the 51 saints on the left (but not on the right) is looking away! This is obviously a deliberate decision on the part of Giotto. Was it a self-portrait? Or a portrait of one of the Baroncelli family who commissioned the work? It's an intriguing mystery. There's an old Japanese proverb: if you want to break the rules of symmetry, you first have to understand them. I hope that today I've given you a bit more of that understanding.



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## Further Reading

- The classic text on symmetry is Hermann Weyl's *Symmetry*, published by Princeton University Press, ISBN 978-0691173252.
- For a deeper look, try the beautifully illustrated 2008 book *The Symmetries of Things* by John H. Conway, Heidi Burgiel, and Chaim Goodman-Strauss. ISBN 978-1568812205
- http://www.neverendingbooks.org/the-scottish-solids-hoax is an excellent article on Neolithic "platonic solids". See also the Encyclopaedia of Polyhedra https://www.georgehart.com/virtual-polyhedra/vp.html by George Hart (no relation!) which has many resources including a blog about the Neolithic stone balls.

## Images shown in the lecture

Anything not mentioned below is either a diagram drawn by me or in the public domain with no usage restrictions.

- The photograph of Palladio's Villa Rotonda a Vicenza is by Marco Bagarella, available on Wikipedia.
- Leonardo Da Vinci's platonic solid drawings are from Luca Pacioli's 1509 book *De divina proportione*. The book is in the public domain, and the images are from the Pennsylvania State University Libraries facsimile edition of *De divina proportione* published by Silvana Editoriale in 1982. They are available online at https://www.maa.org/press/periodicals/convergence/leonardo-da-vincis-geometric-sketches-introduction.
- Chris Watson's webpage https://www.tessellationart.com/ has lots of examples of his work, and he's releasing a free book, the Little Book of Tessellation, for you to try your own tessellation art. https://www.tessellationart.com/thelittlebookoftessellation/. His design "Fish Orange" is shown with permission.
- Zarah Hussain's work has been featured in many exhibitions, including 2017's *Numina* at the Barbican Centre in London and the recent *Breath* exhibition at the Peabody Essex Museum in Massachusetts (until April 2022). If you are based in London, you can pop to Vernon Road in Walthamstow to see a mural of her work for the Wood St. Walls project. https://www.zarahhussain.co.uk/.