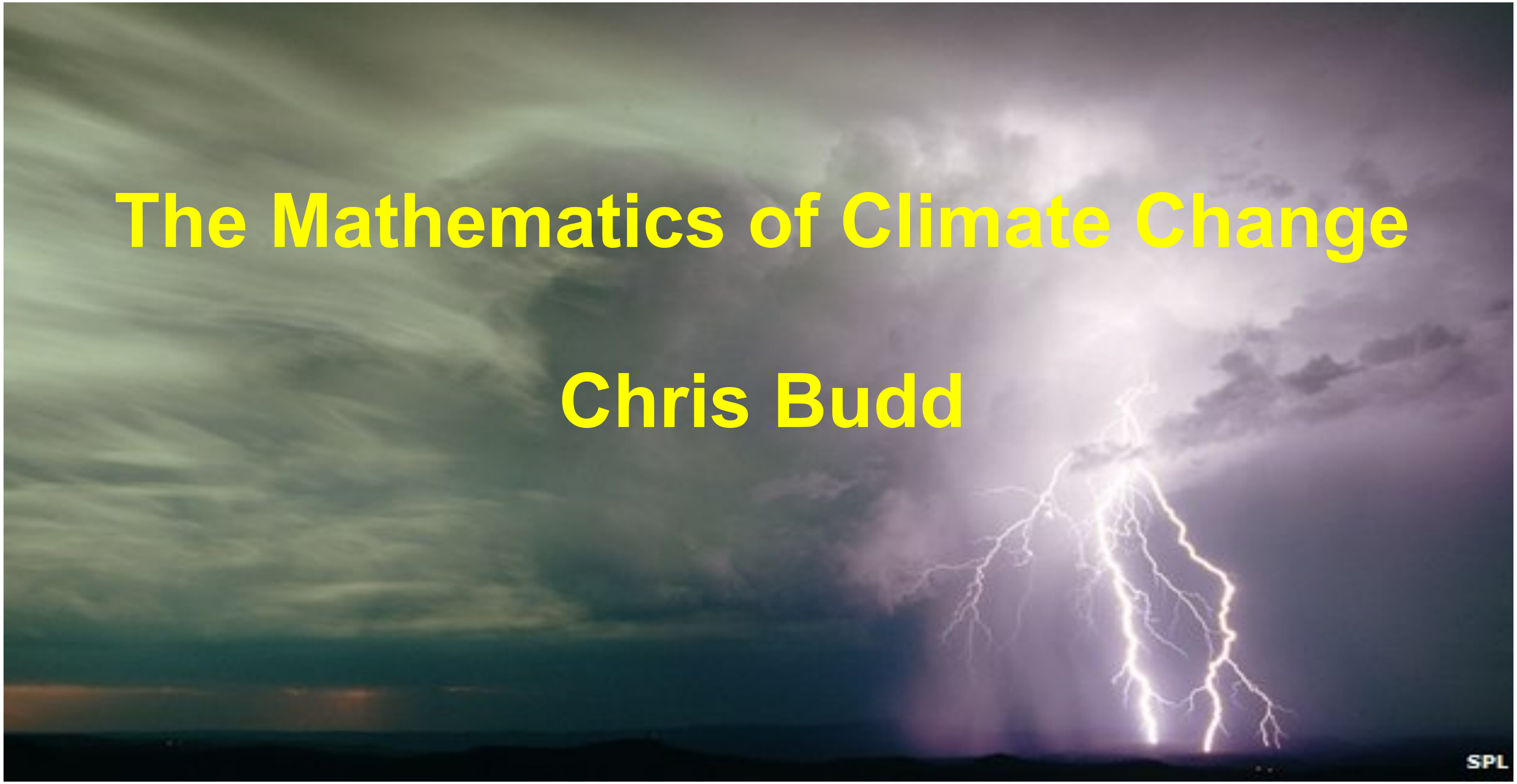
The Mathematics of Climate Change

Chris Budd



GRESHAM COLLEGE

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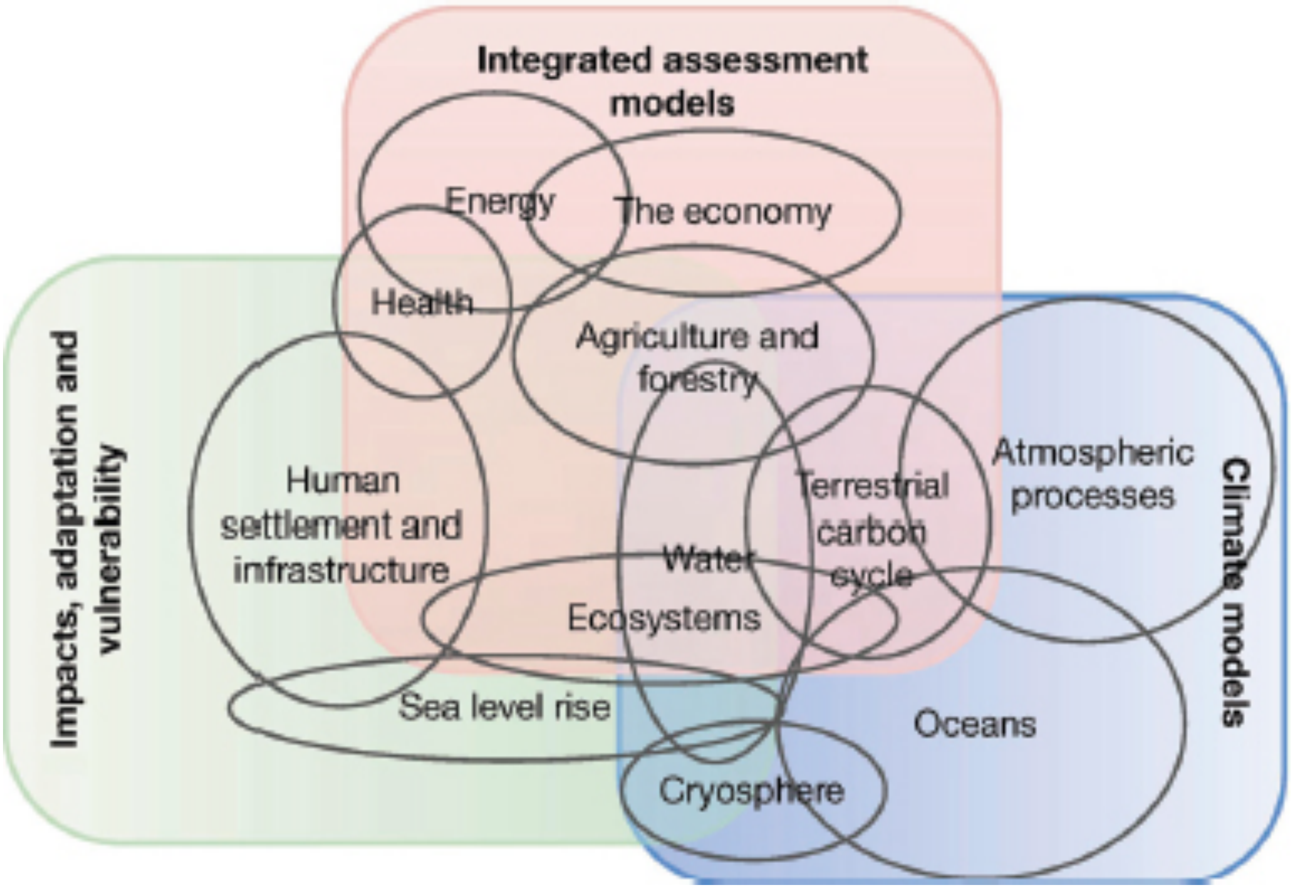
UNIVERSITY OF BATH

Human made climate change is:

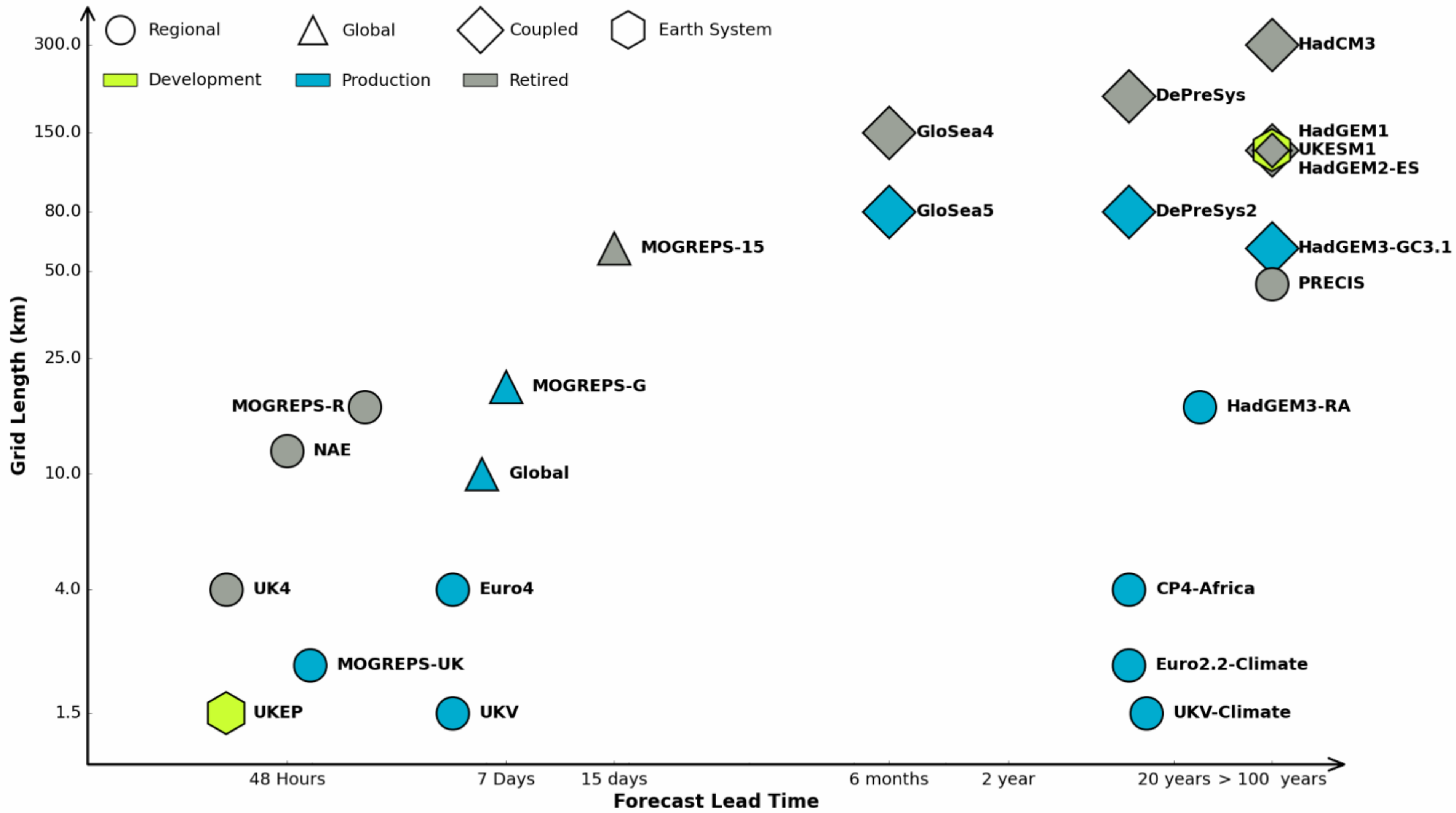
- Controversial
- Important
- A meeting place of scientists, mathematicians and policy makers

Climate modelling is hard, uncertain, and lacks good data

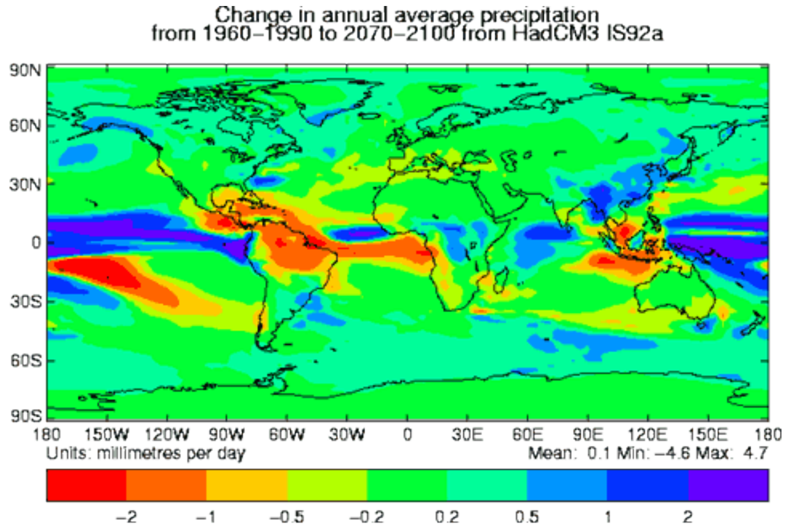
Modern Global Climate Models GCMs are highly complex with billions of degrees of freedom and take large parallel computers to run Used to inform government policy!



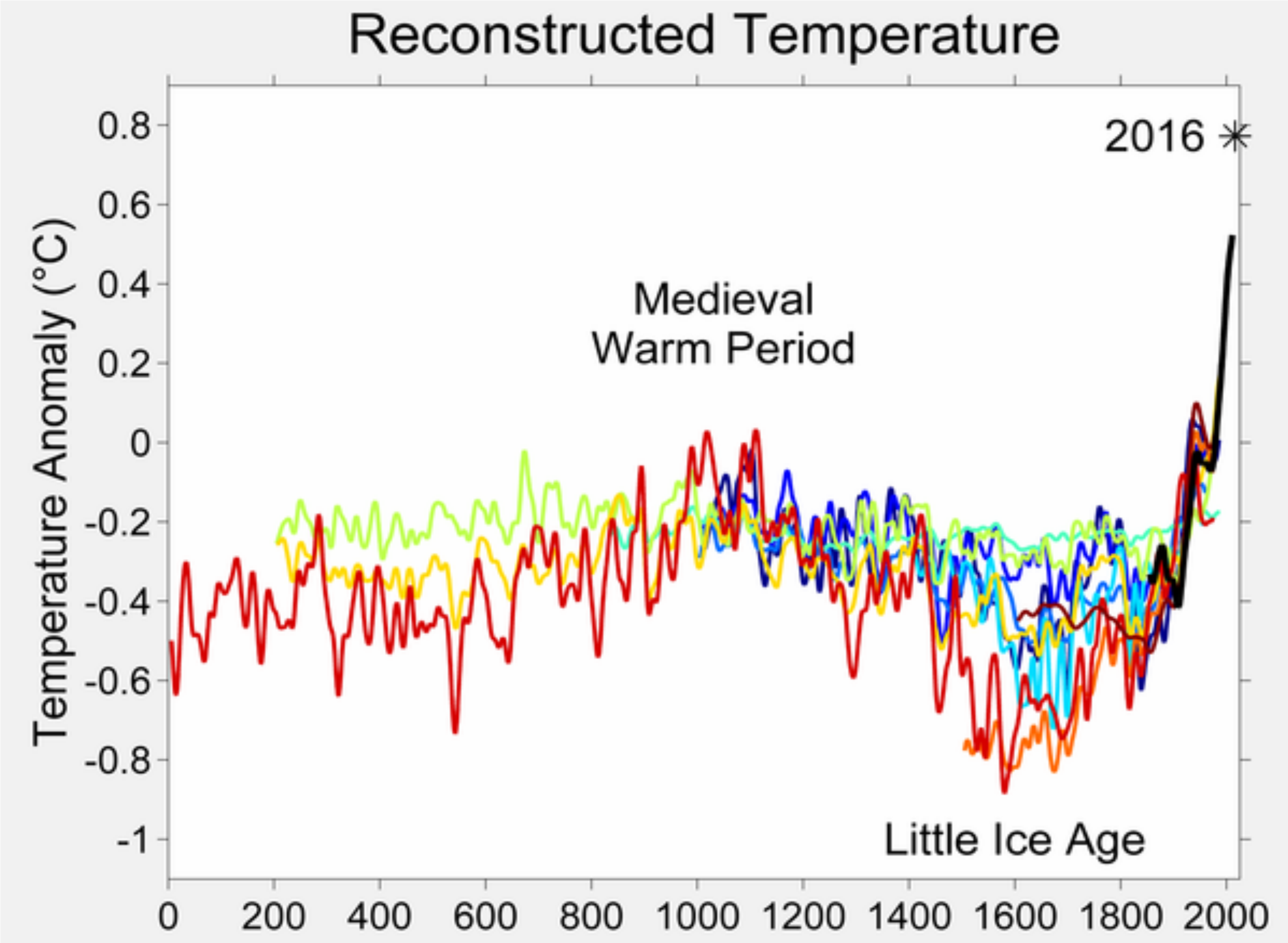
Whole hierarchy of weather and climate models

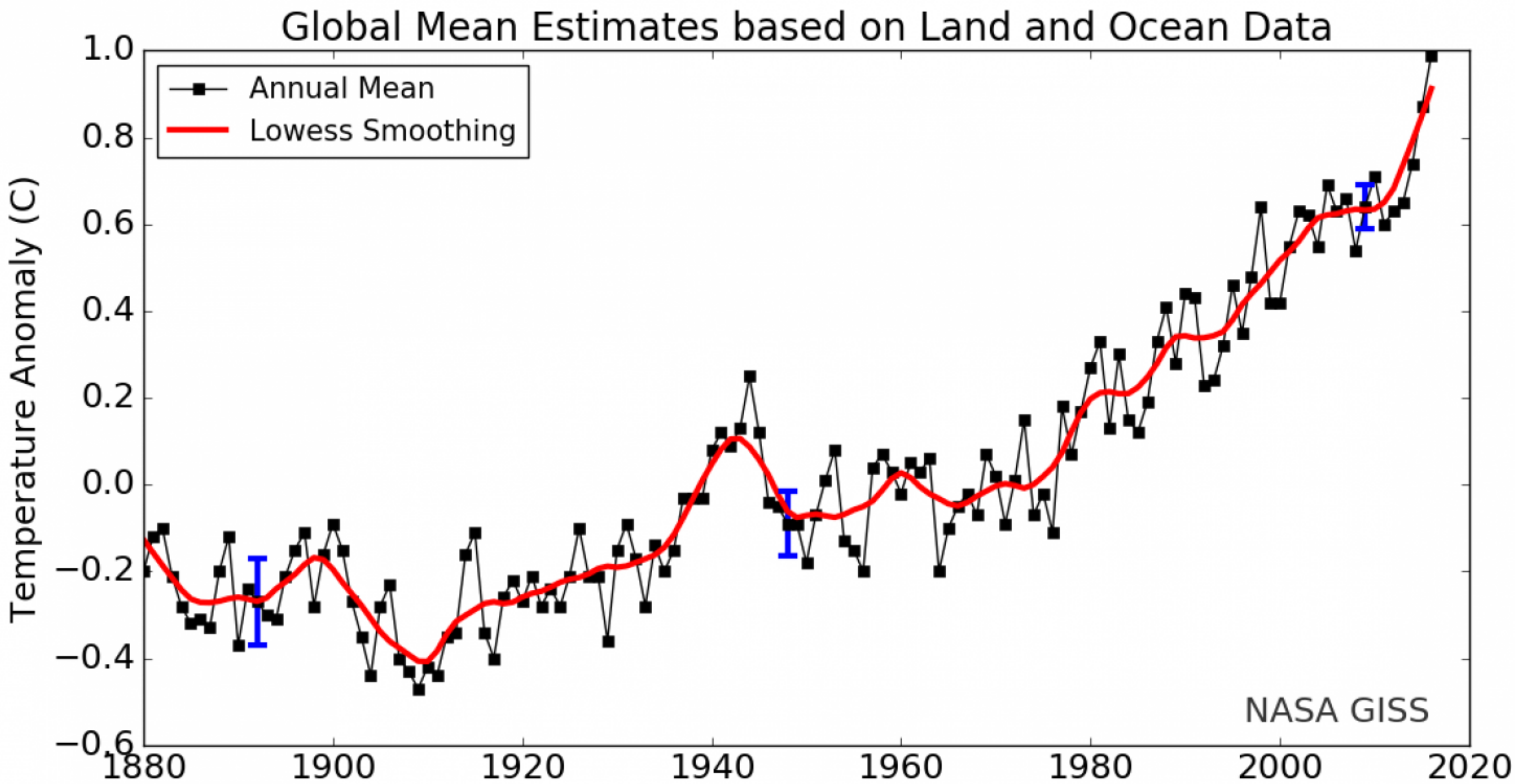


The data: five 'official' indicators of climate change

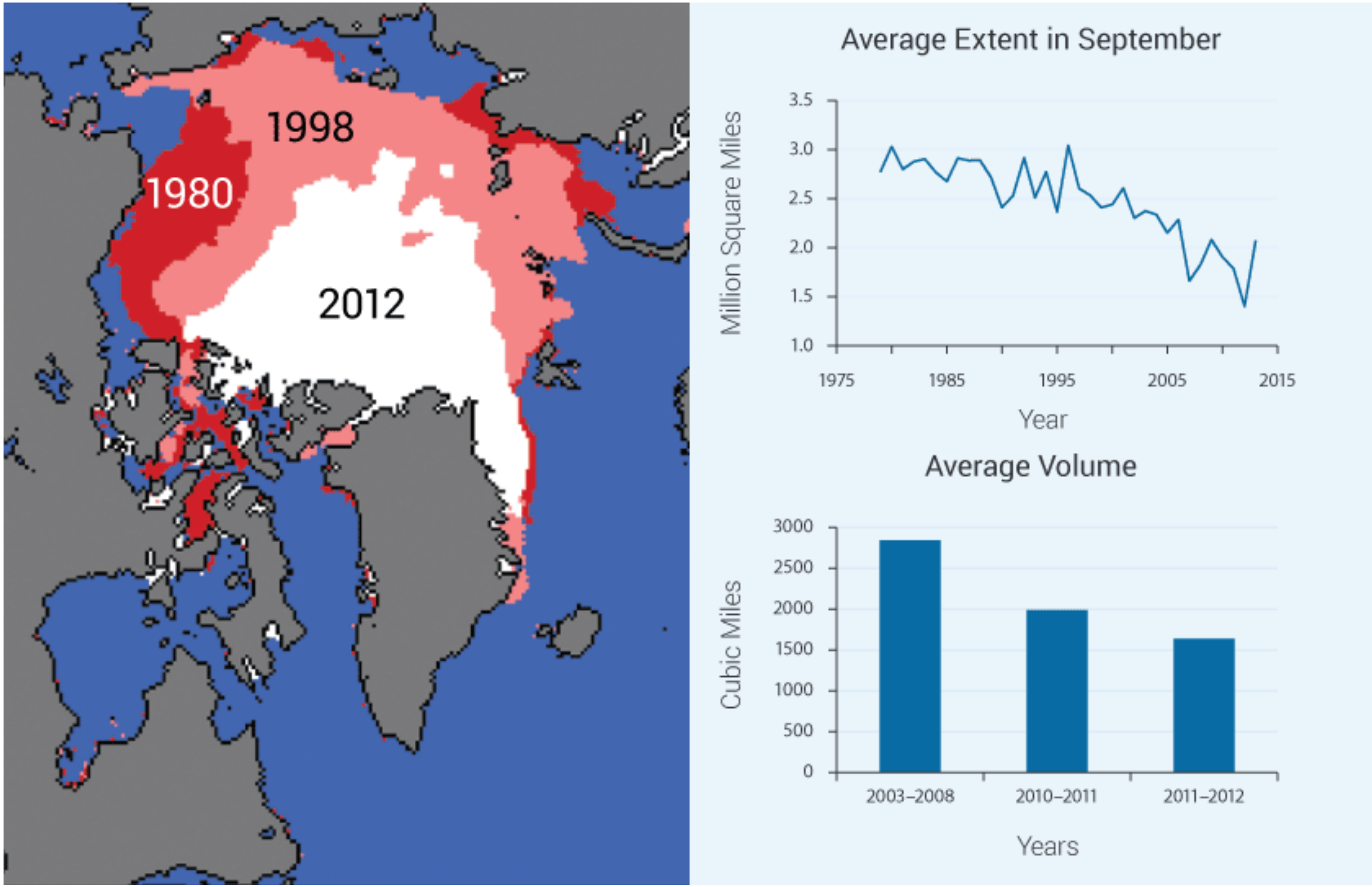


1. Increasing temperatures




Global Mean Estimates based on Land and Ocean Data
Annual Mean
Lowess Smoothing
Temperature Anomaly (C)
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
1880
1900
1920
1940
1960
1980
2000
2020
NASA GISS

2. Arctic Sea Ice Loss



Nasa’s conclusion

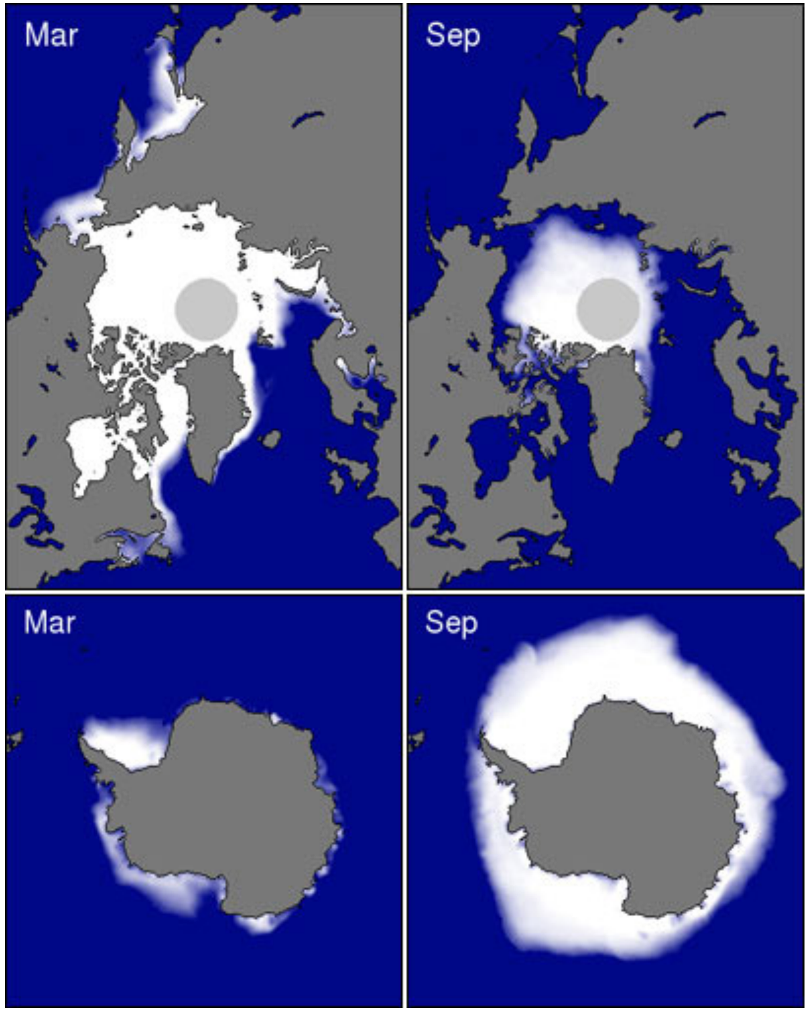
Arctic sea ice has become thinner by around 43% over the last 25 years

… and this trend is continuing

But … from a New Zealand perspective

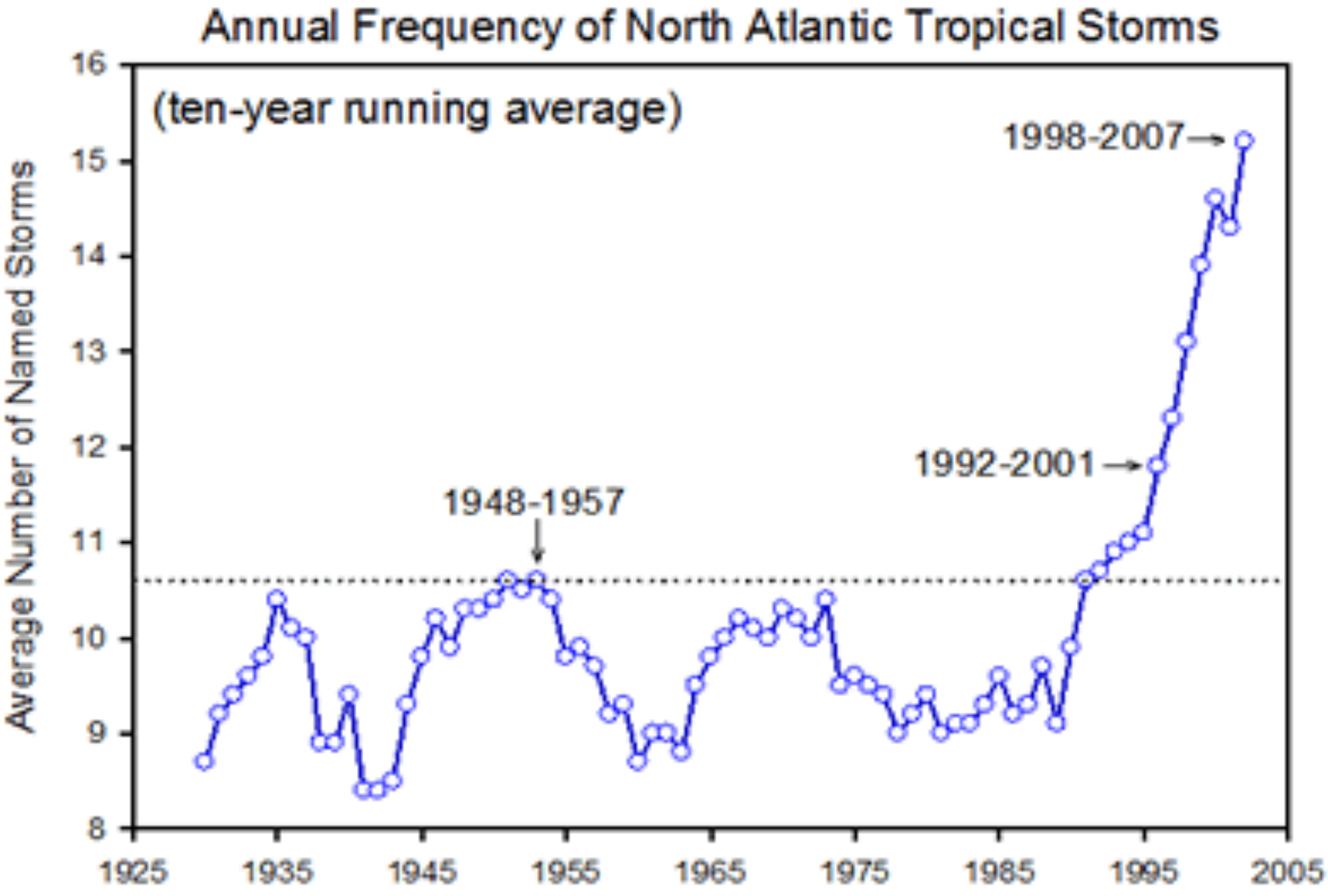
Antarctic sea ice is actually increasing!

Although land ice is decreasing

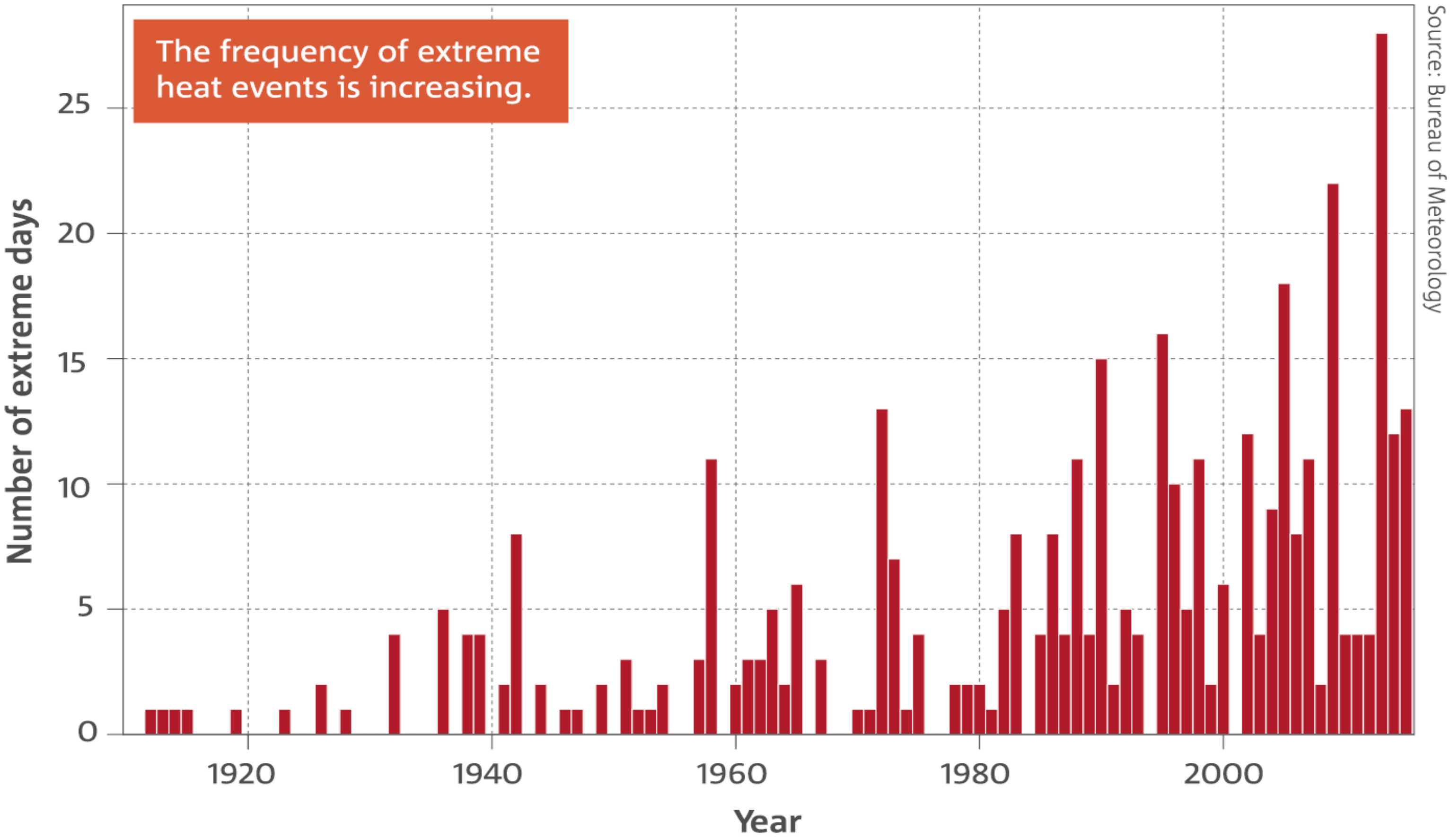


3. More Extreme Weather Events

2014 St Valentines Day Storm

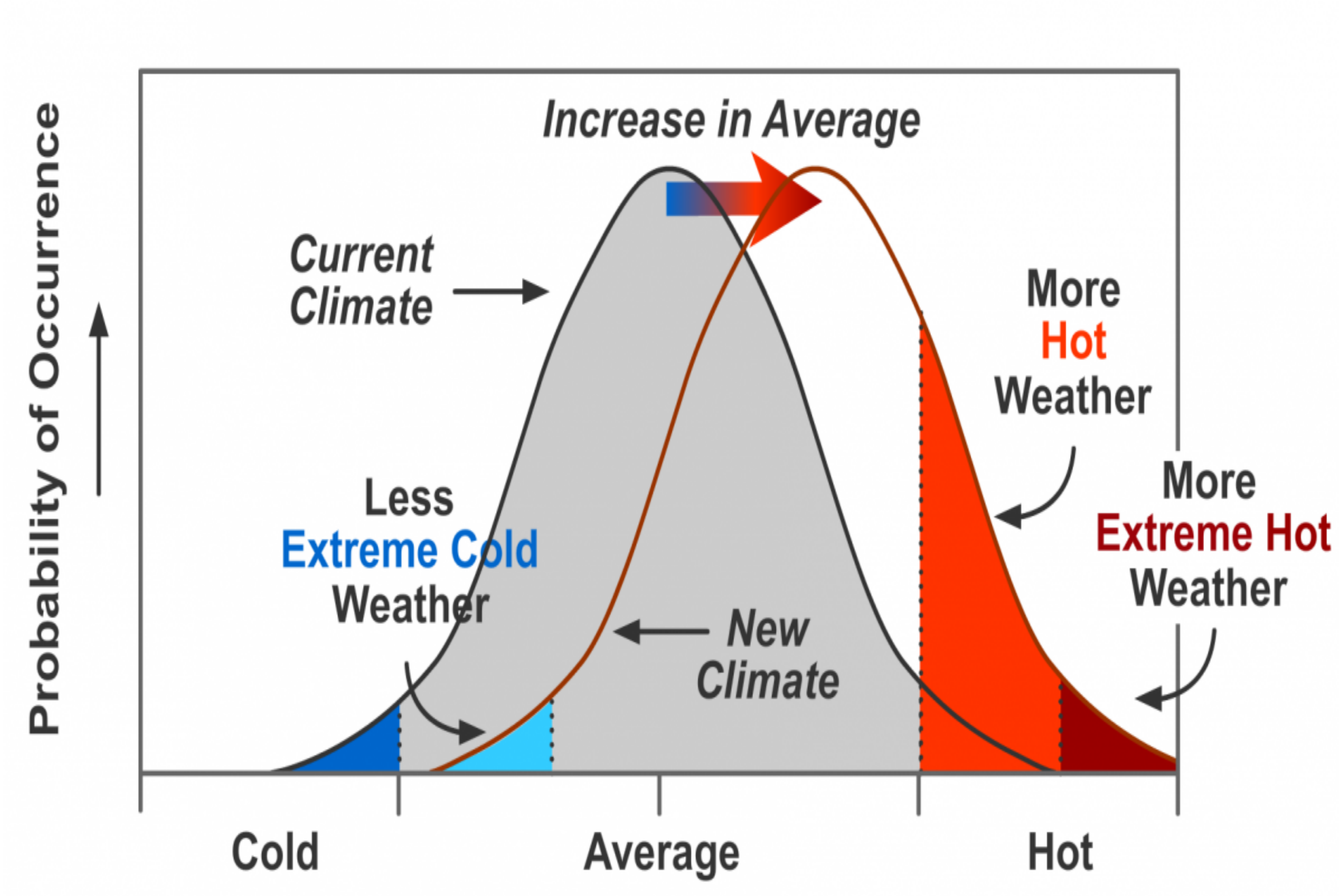


Annual Frequency of North Atlantic Tropical Storms
(ten-year running average)
Average Number of Named Storms
1998-2007
1992-2001
1948-1957
16
15
14
13
12
11
10
9
8
1925
1935
1945
1955
1965
1975
1985
1995
2005



The frequency of extreme heat events is increasing.
Number of extreme days
25
20
15
10
5
0
1920
1940
1960
1980
2000
Year
Source: Bureau of Meteorology

This is mathematically consistent with global warming



4. Sea Level Rise

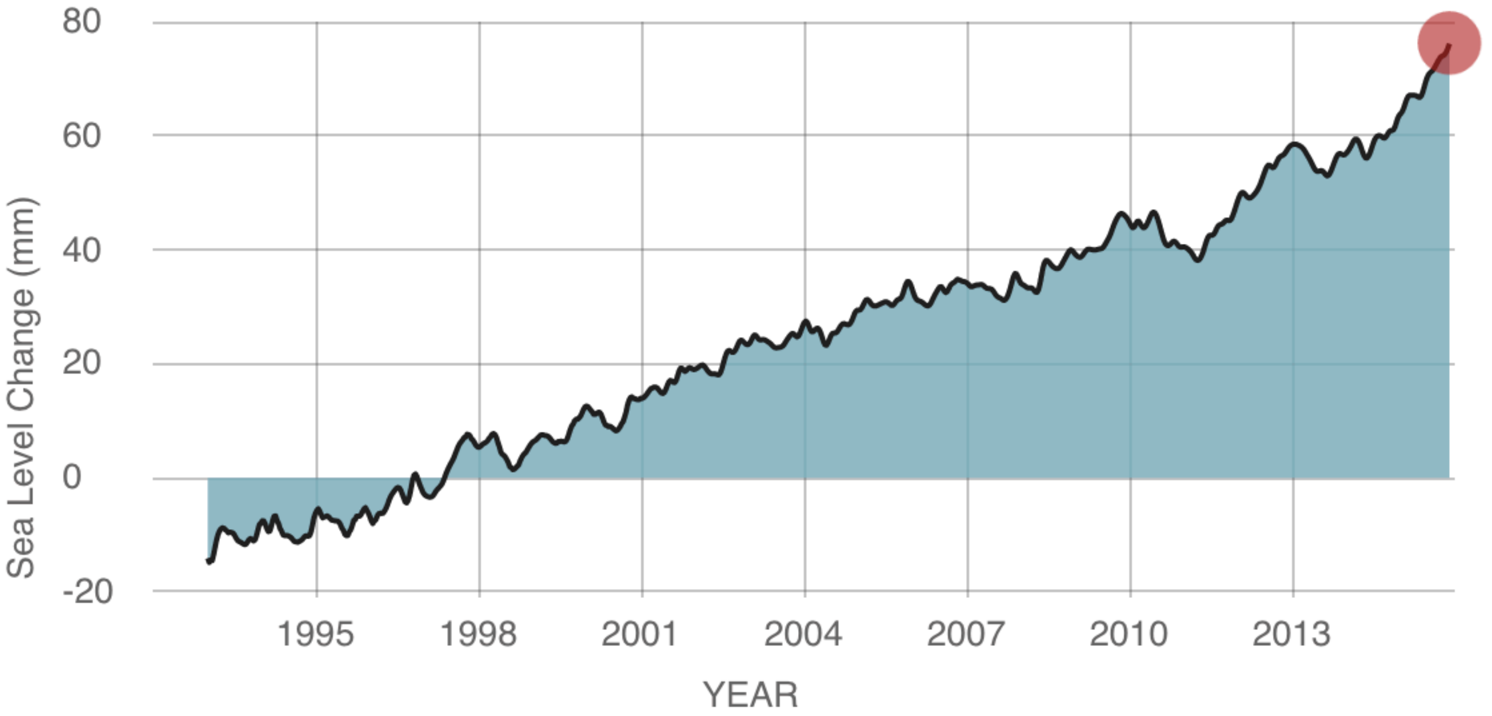
SATELLITE DATA: 1993-PRESENT

Data source: Satellite sea level observations.
Credit: NASA Goddard Space Flight Center

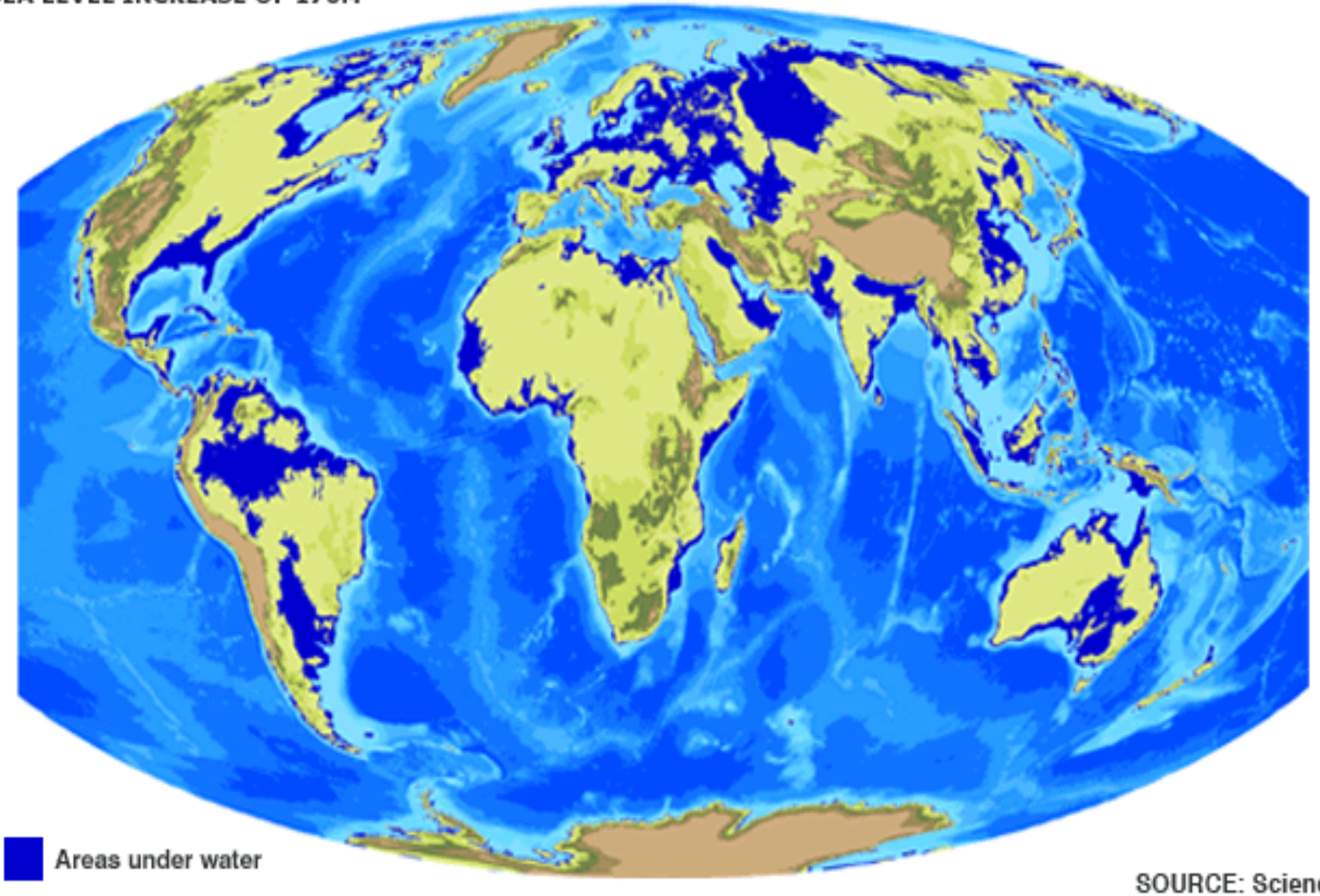
RATE OF CHANGE

↑ 3.41

mm per year

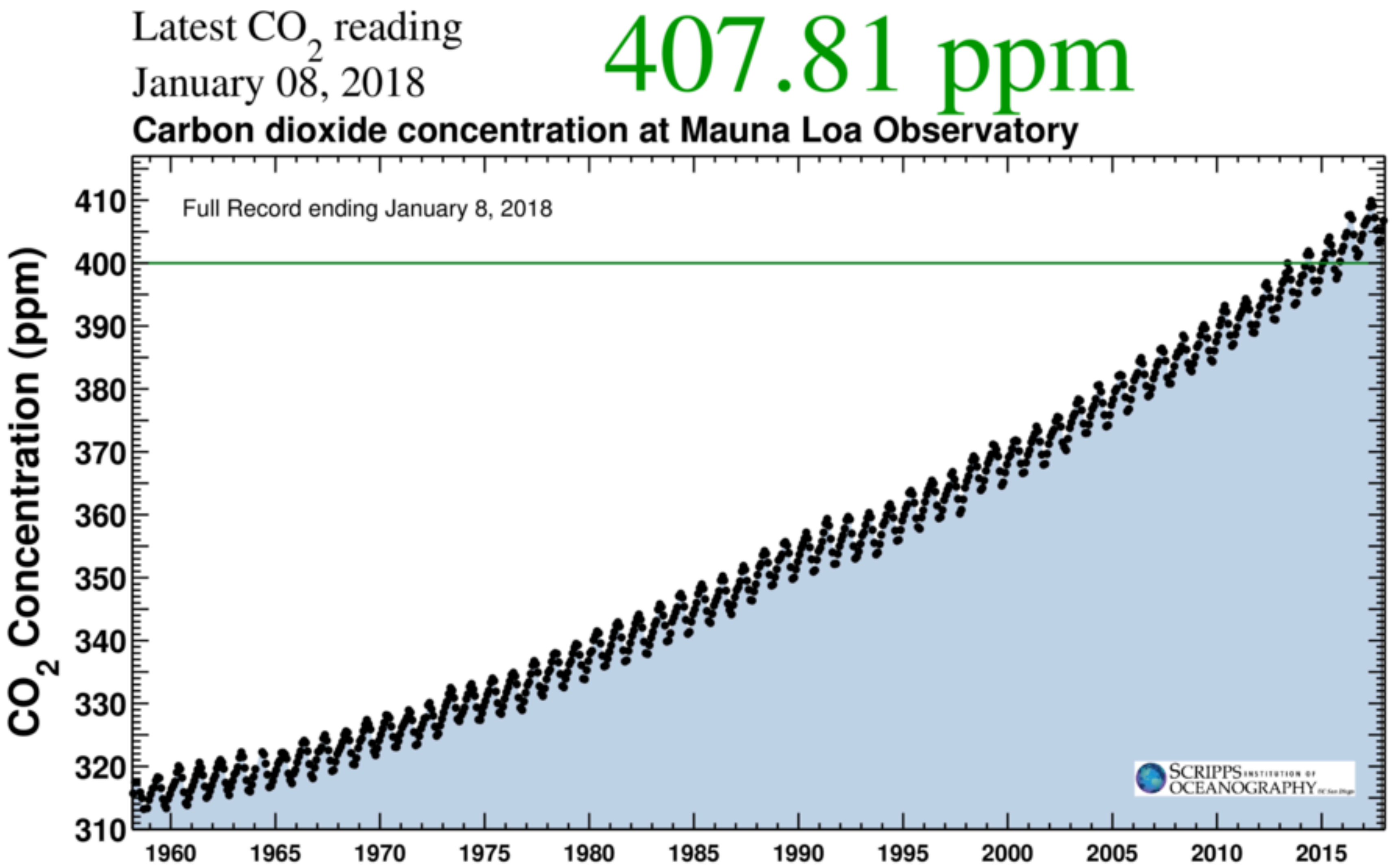


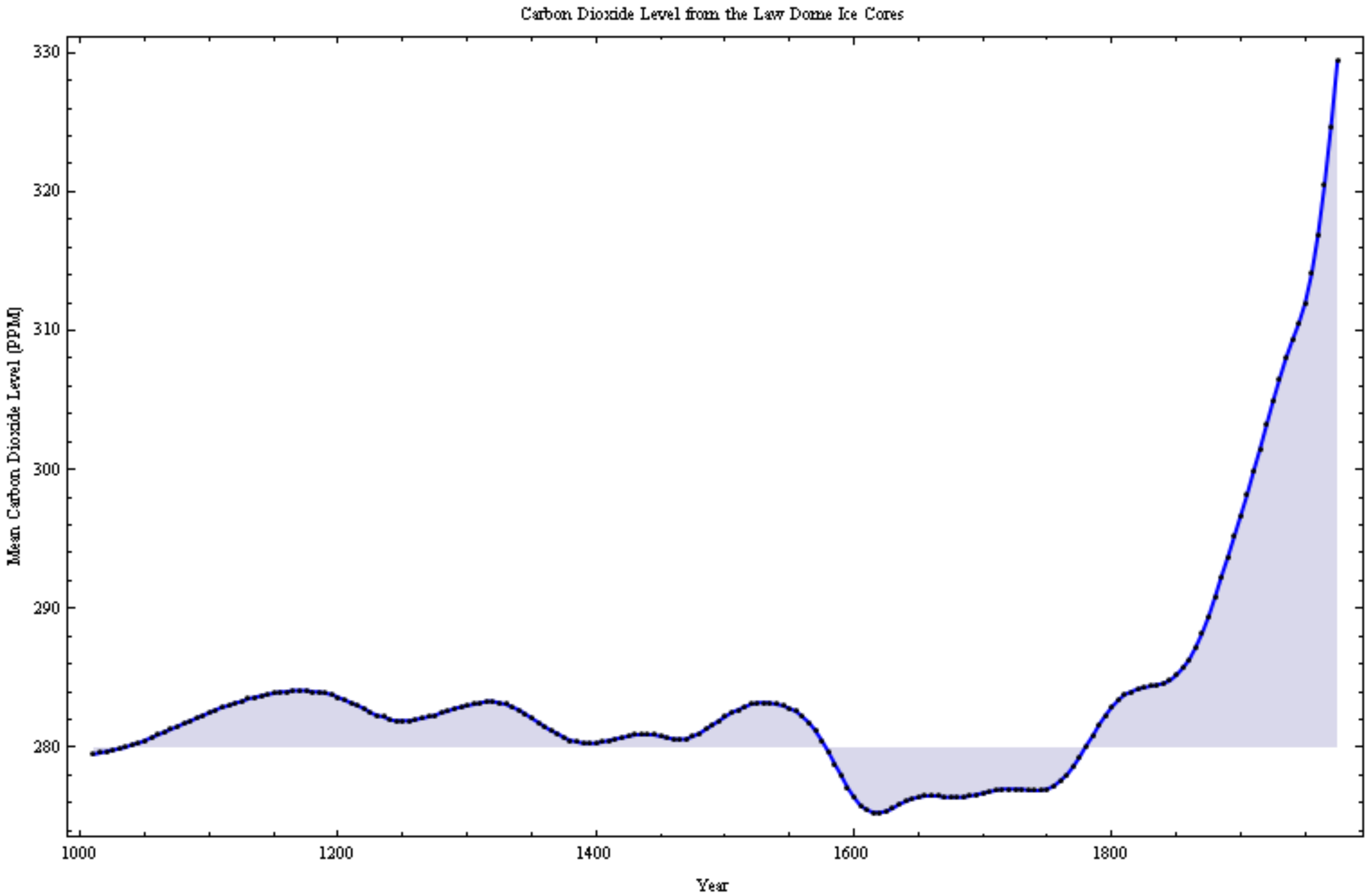
SEA LEVEL INCREASE OF 170M



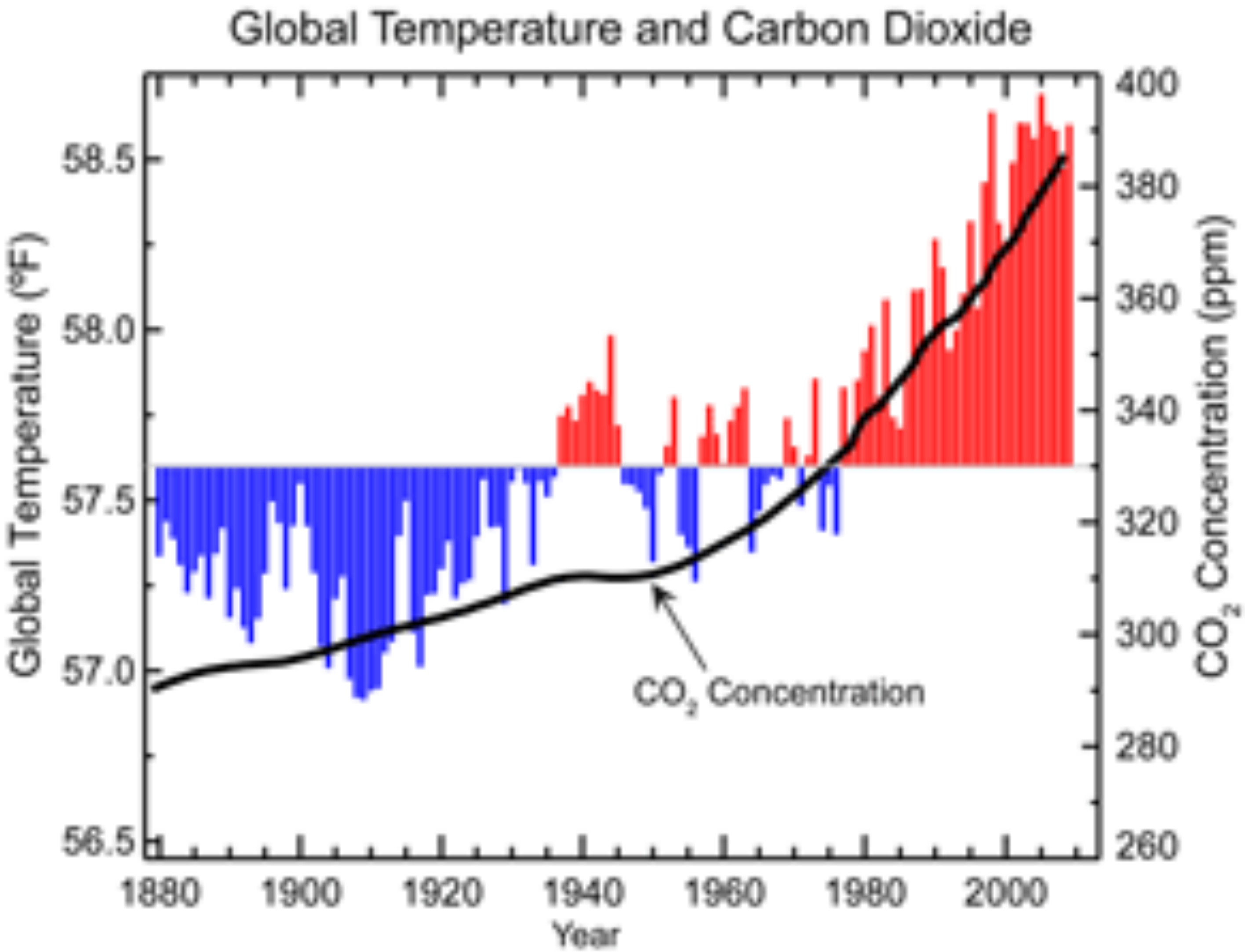
SOURCE: Science

5. Increase in Atmospheric Carbon Dioxide




Carbon Dioxide Level from the Law Dome Ice Cores
Mean Carbon Dioxide Level (PPM)
330
320
310
300
290
280
1000
1200
1400
1600
1800
Year

There is a connection with rising temperatures



Not everyone agrees!

"This is nonsense….there has been no statistically significant global warming for at least a decade”.”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DAILY EXPRESS

How to eat, drink and stay healthy

yourhealth starts page 38

12 DAYS OF CHRISTMAS CHAOS IN BA STRIKE

100 REASONS WHY GLOBAL WARMING IS NATURAL

‘No proof that human activity is to blame’

SIMON COWELL

I’ve made millions but I have such dark moods

Watts up with that

And maybe it’s all a big conspiracy!

ClimateGate

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SAVE 90P A WEEK ON YOUR DAILY & SUNDAY EXPRESS

DAILY EXPRESS

Clooney's amazing mother

FREE £5 SPEND AT WHSmith FOR EVERY READER

Iran threatens serious action against sailors

THE BIG CLIMATE CHANGE 'FRAUD'

We are not to blame says top scientist... It's a con to raise tax

IT'S FURRY NICE TO MEET YOU

Why is climate science so hard?

It is difficult to predict anything, especially about the future! *Niels Bohr*

Some reasons for the uncertainty

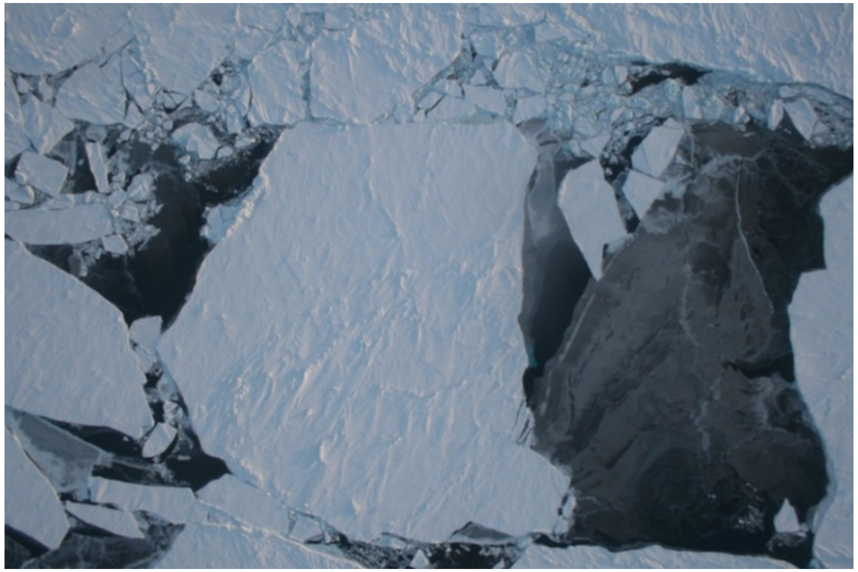
Statistical variation in dodgy data

Chaos

Complexity of the system

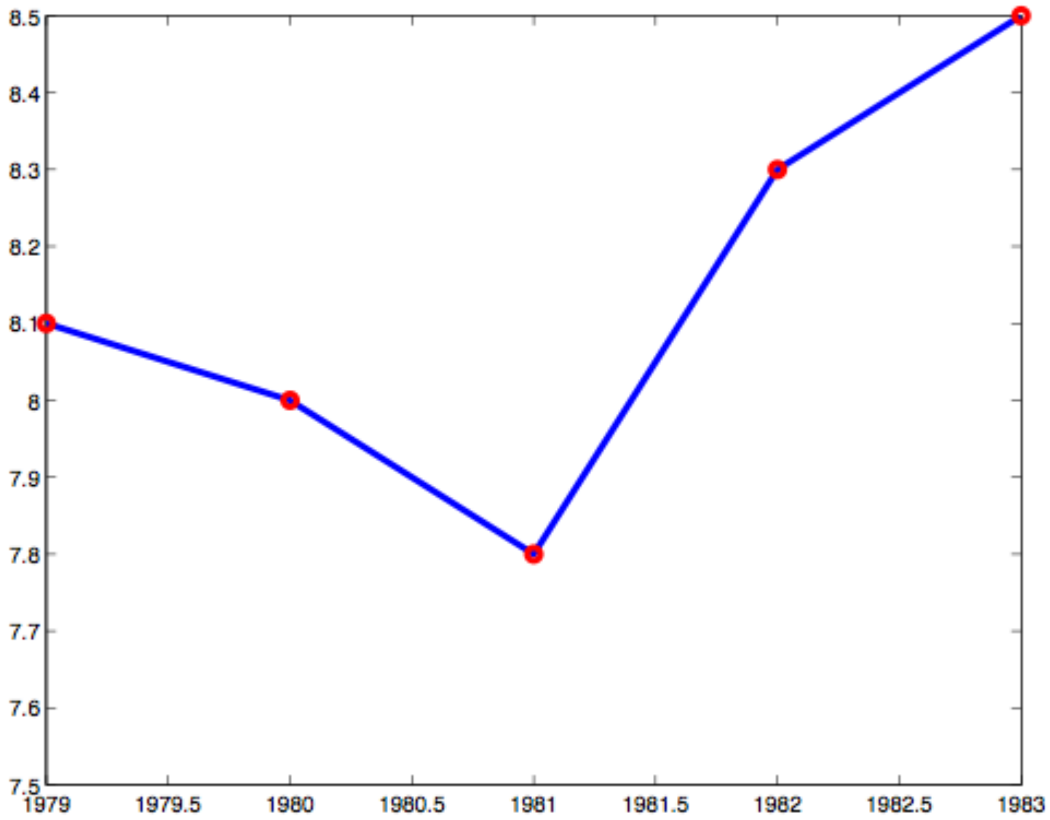
Distinguishing between natural and human made variation

Example: How much Arctic Sea Ice Is There?



NASA: National Snow and Ice Data Center NSIDC

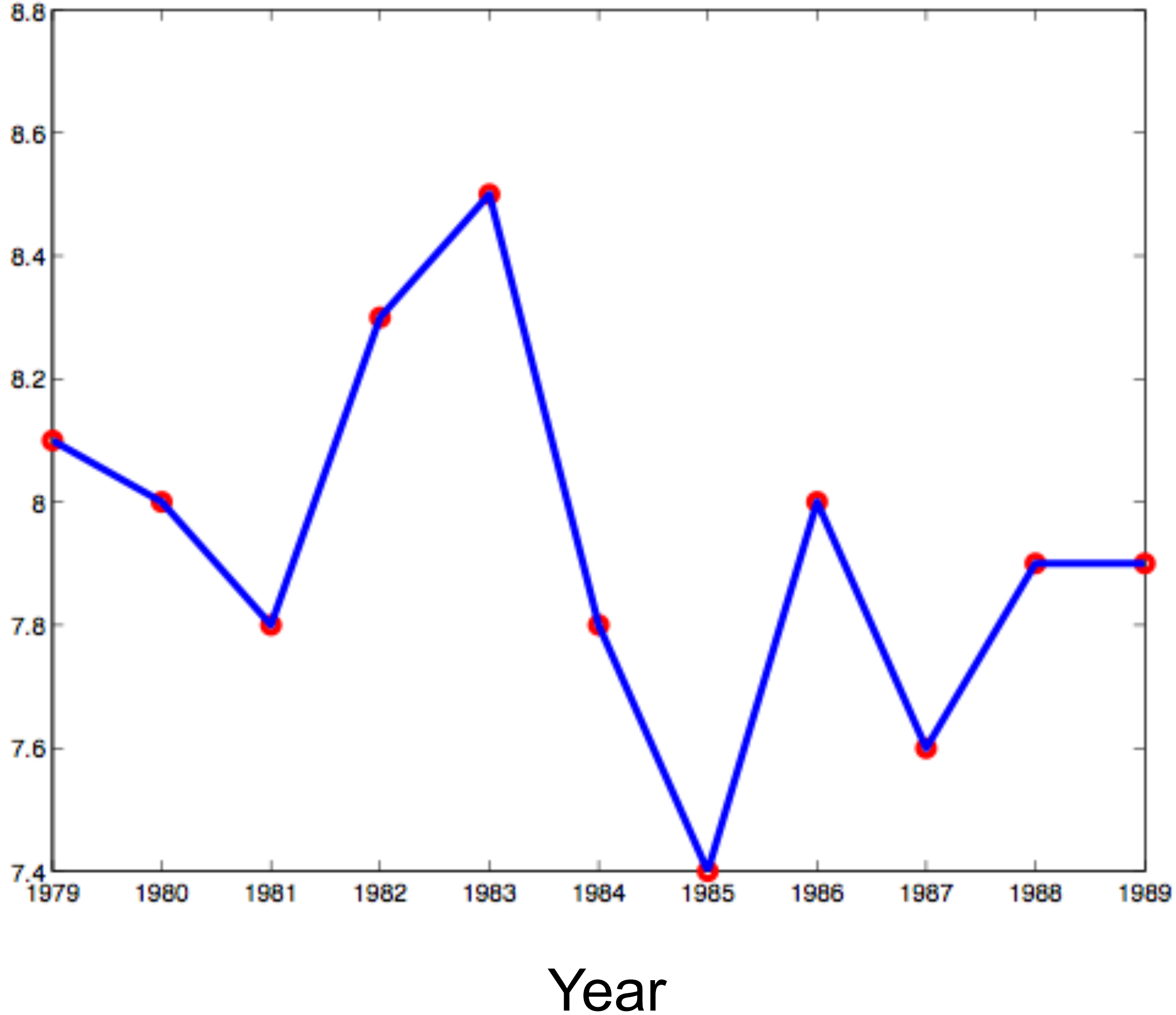
Summer sea ice in millions of square km



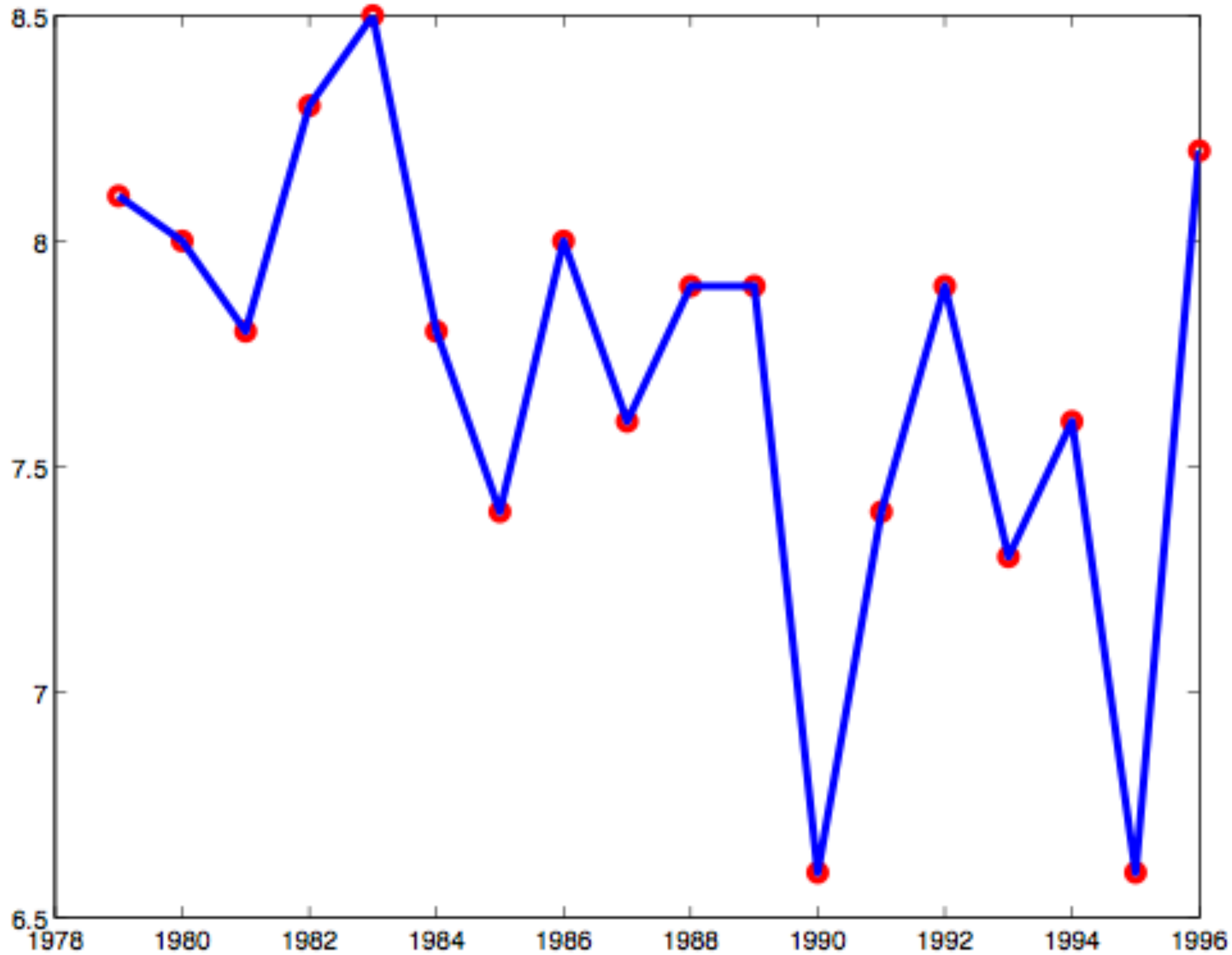
Year

What happens next?

How much Ice is there in millions of square km?

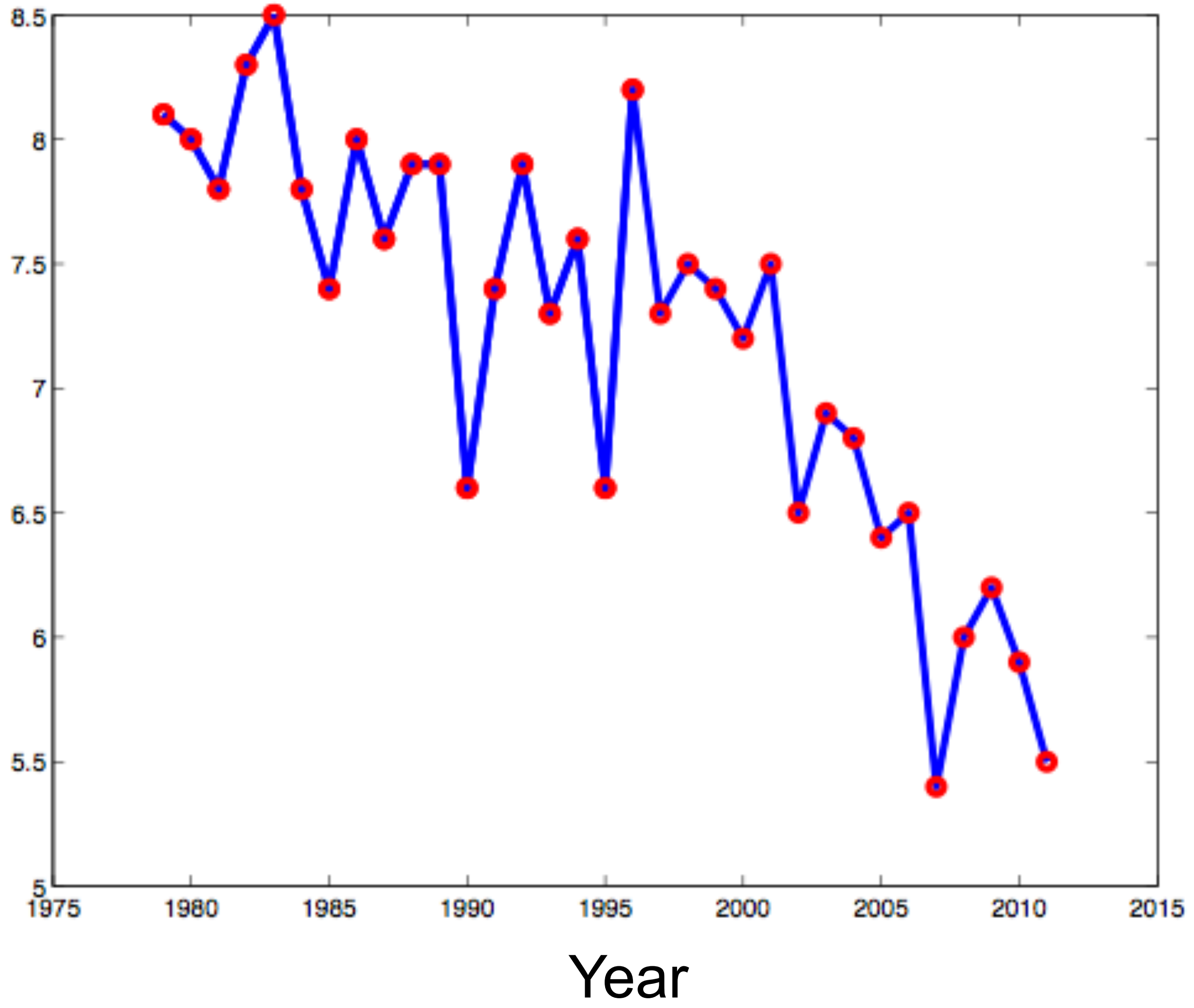


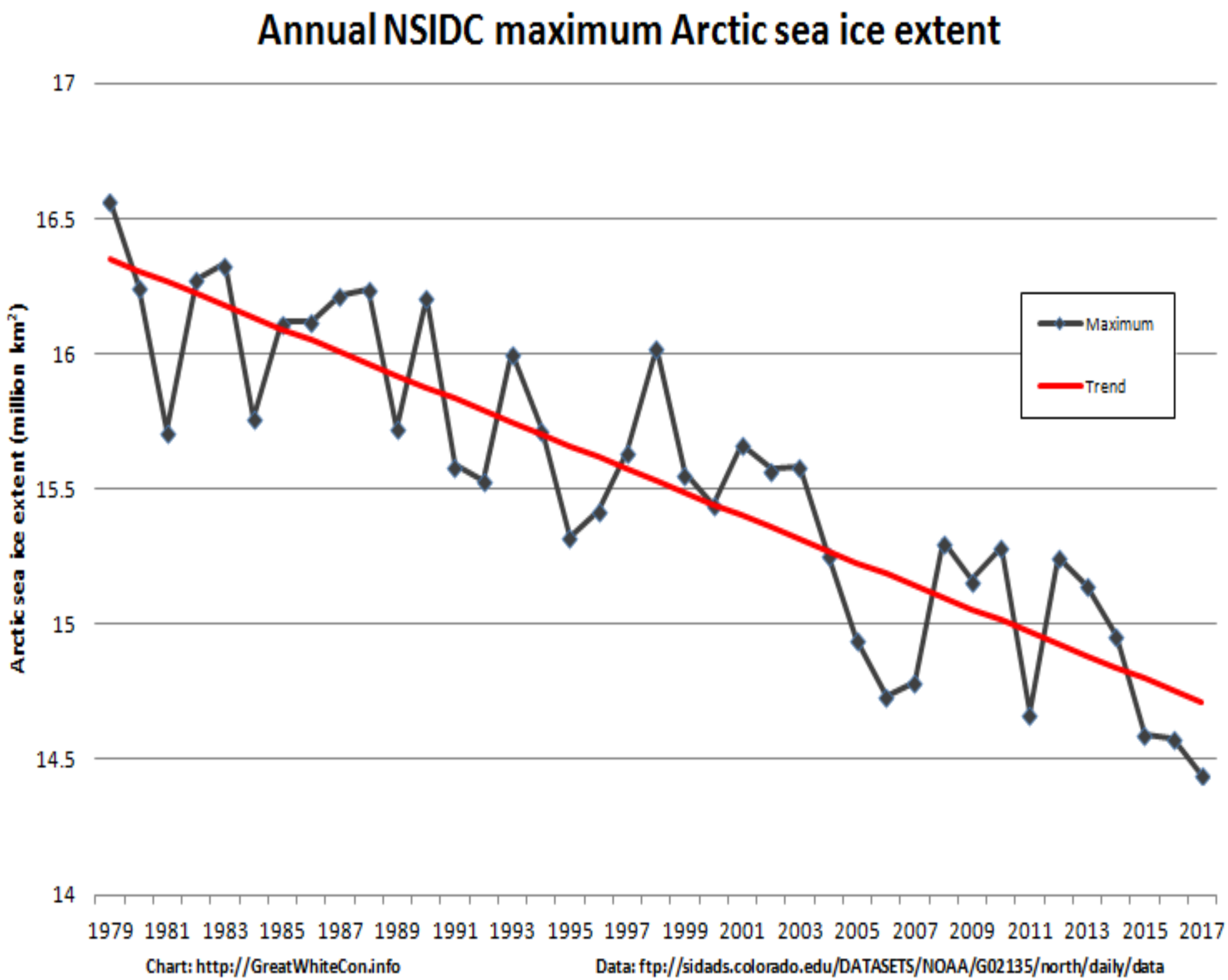
How much Ice is there in millions of square km?



Year

How much Ice is there in millions of square km?

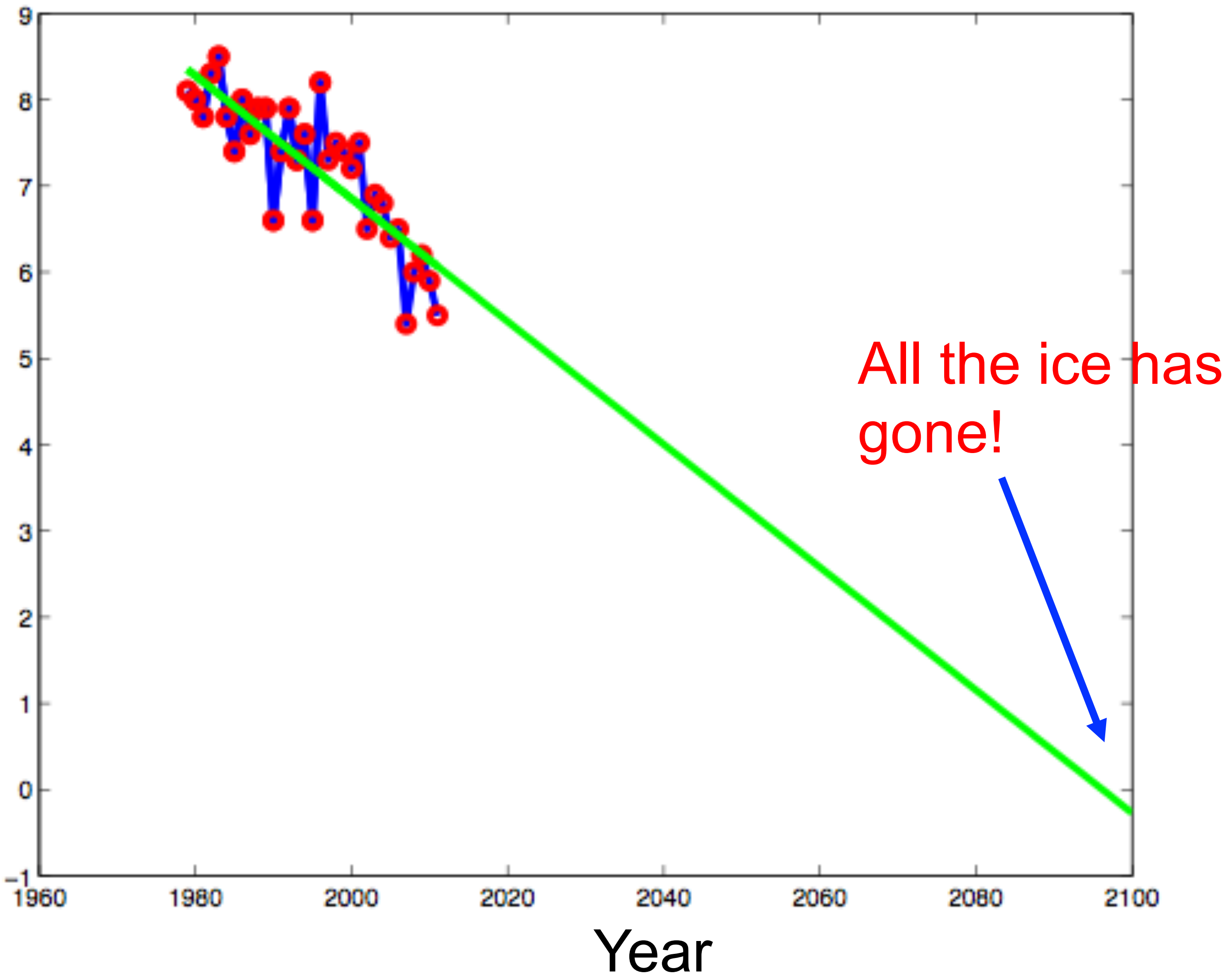




Annual NSIDC maximum Arctic sea ice extent
17
16.5
16
15.5
15
14.5
14
Arctic sea ice extent (million km²)
Maximum
Trend
1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
Chart: http://GreatWhiteCon.info
Data: ftp://sidads.colorado.edu/DATASETS/NOAA/G02135/north/daily/data

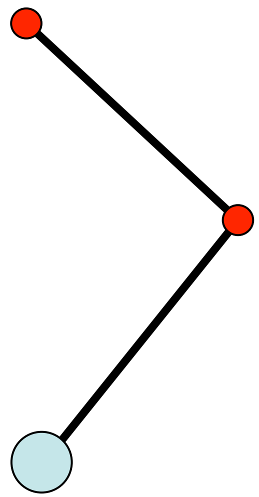
Future prediction???

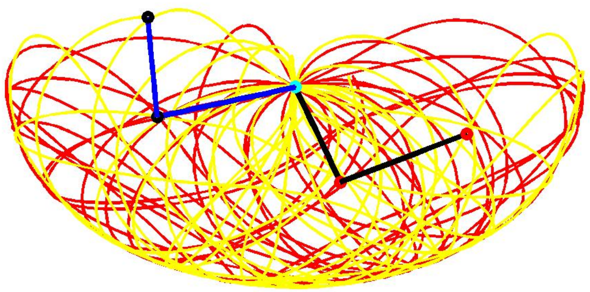
How much Ice is there in millions of square km



Chaos theory tells us that there is …

A limit to our scientific understanding of the future





Motion can be Chaotic and unpredictable

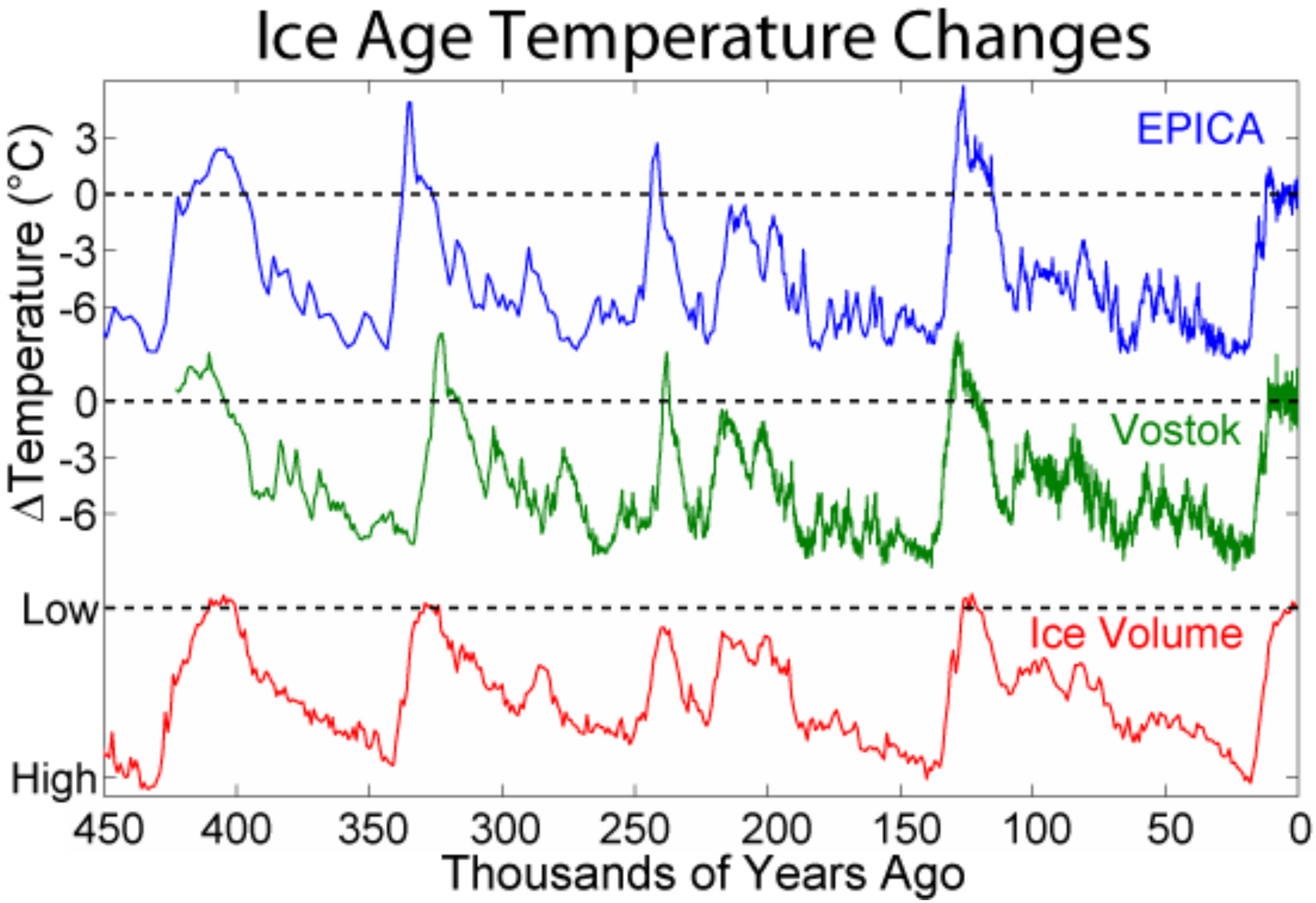
Even if we understand something we can't always predict it with certainty!!!!!!

3. But can we really tell the future from the past?

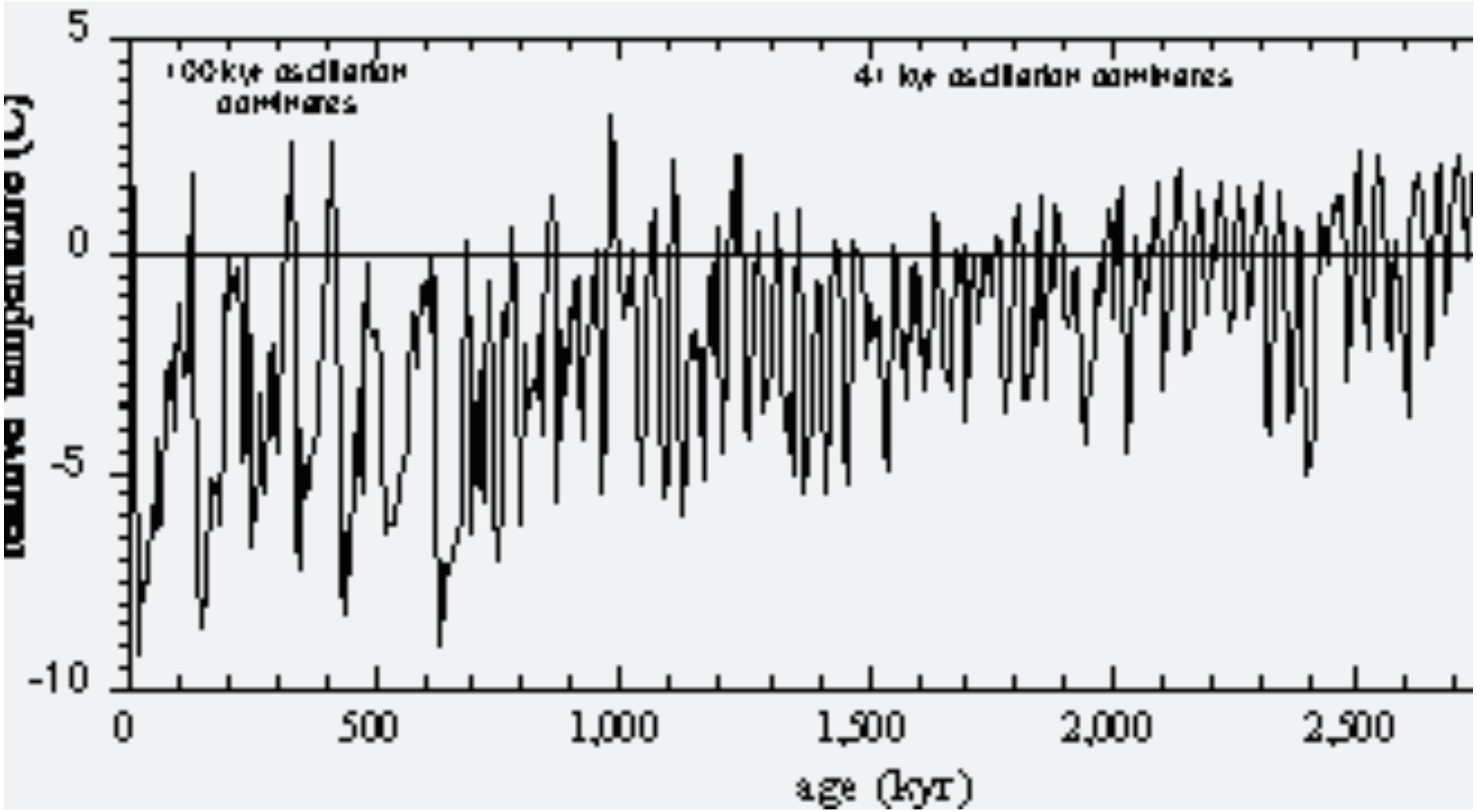
Surely climate has been changing a lot over the past!

Isn’t what we see now just part of that natural change?

What happened in the last half a million years?

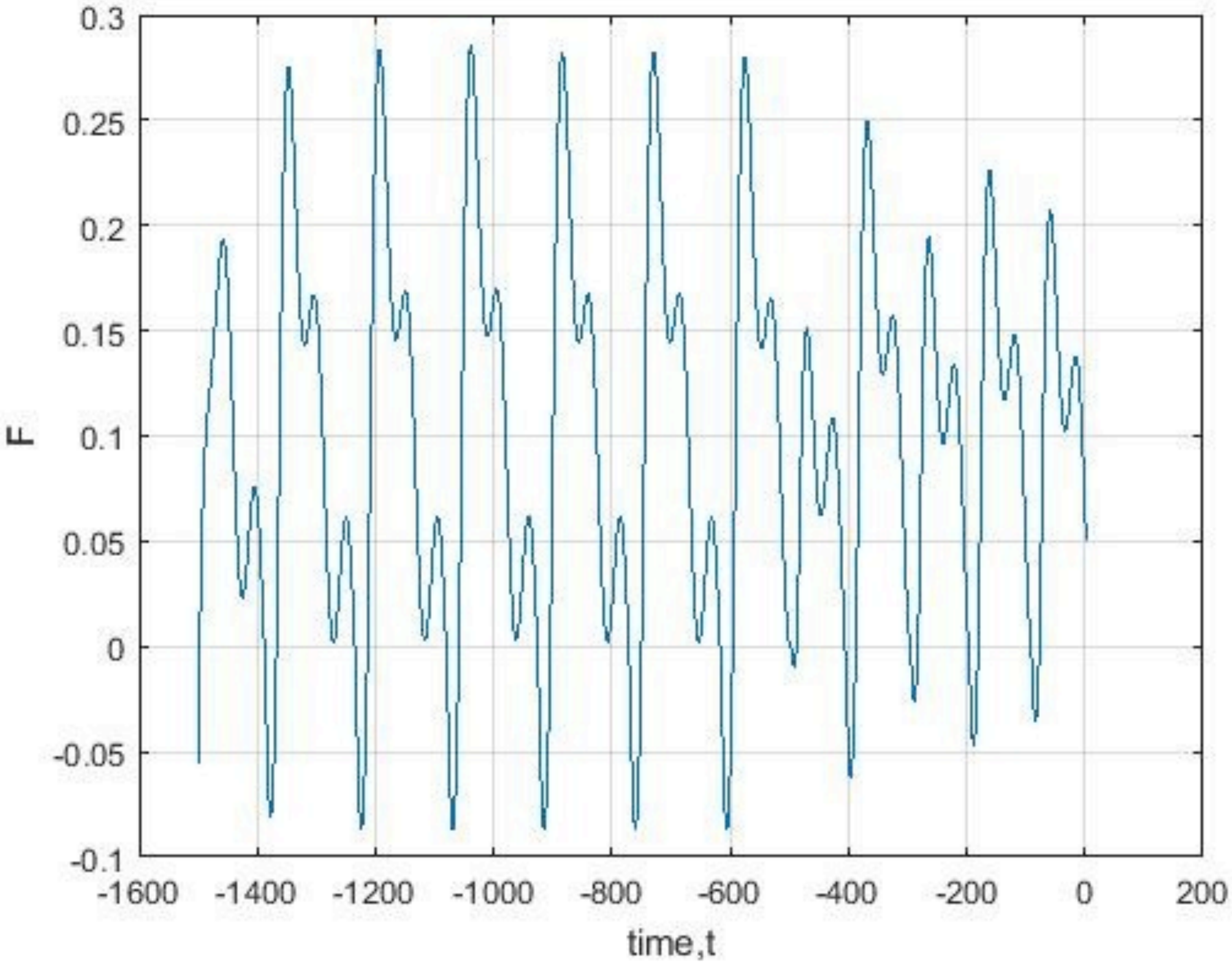


Appearance of large temperature increases in a short time intervals, gradual cooling over 100kyr, with smaller oscillations in the cooler phases.



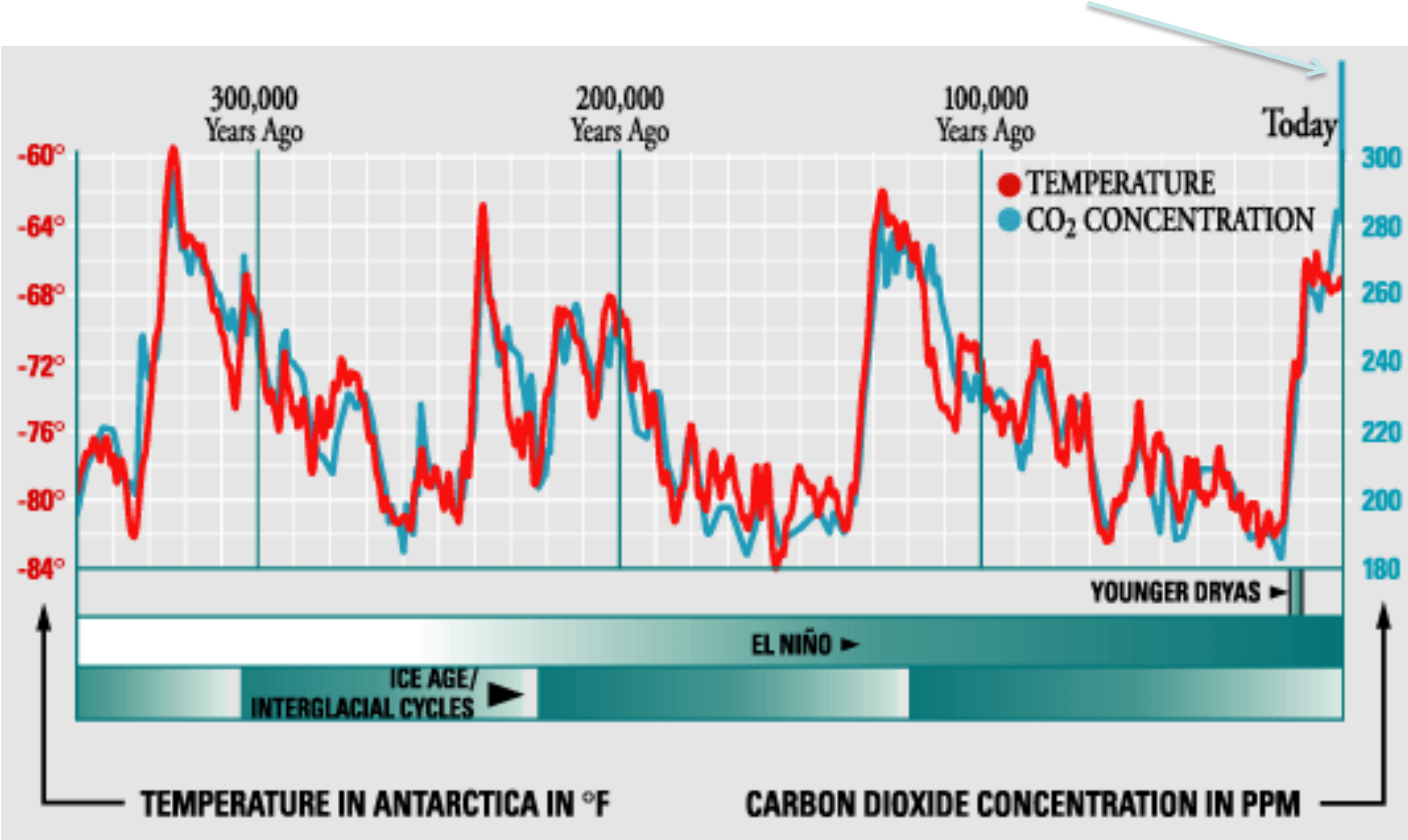
In the last million years: Change in amplitude and period of oscillation from Small 40kyr to large 100kyr about 700kyr ago

Mid Pleistocene Transition (MPT)



MPT simulation by Susan Morupisi (Bath)

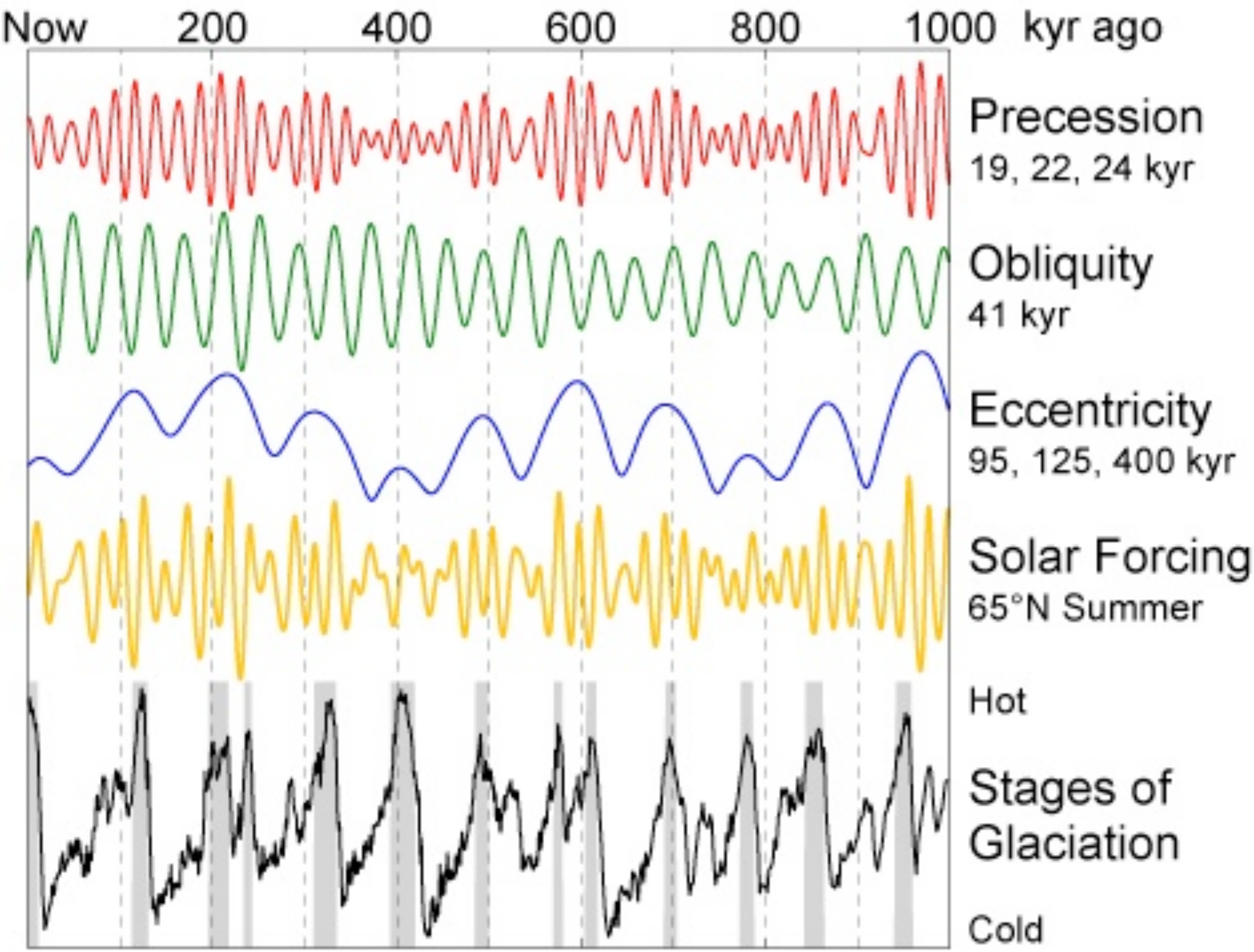
Fast rising hockey stick



Temperature and CO2 and Ice volume in synchronisation!

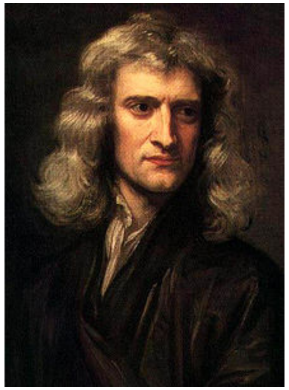
External Solar Input S(t) seems to be partly responsible:

Milankovitch cycles: What happens next?

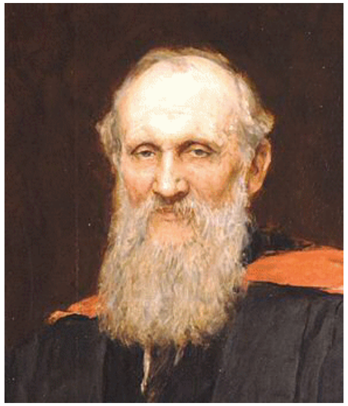


Climate Centres try to work this out via mathematical models

Take laws of physics



Motion



Heat

Turn them into partial differential equations

Solve these on a supercomputer to try to predict the climate

Met Office Hadley Centre

What makes up the climate?

Air Pressure	p
Air Velocity	u
Air/Ocean Temperature	T
Air density	ρ
Moisture	q

Same for the oceans + ice + salt

All affected by:

Solar radiation	S
Earth's rotation	f
Gravity	g

Mountains, vegetation, ice, CO_2, …

Basic equations were derived by Euler and describe the weather

$$\frac{Du}{Dt} + 2f \times u + \frac{1}{\rho}\nabla p + g = \nu \nabla^2 u,$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0,$$

$$C\frac{DT}{Dt} - \frac{RT}{\rho}\frac{D\rho}{Dt} = \kappa_h \nabla^2 T + S_h + LP,$$

$$\frac{Dq}{Dt} = \kappa_q \nabla^2 q + S_q - P,$$

$$p = \rho RT.$$

Motion

Density

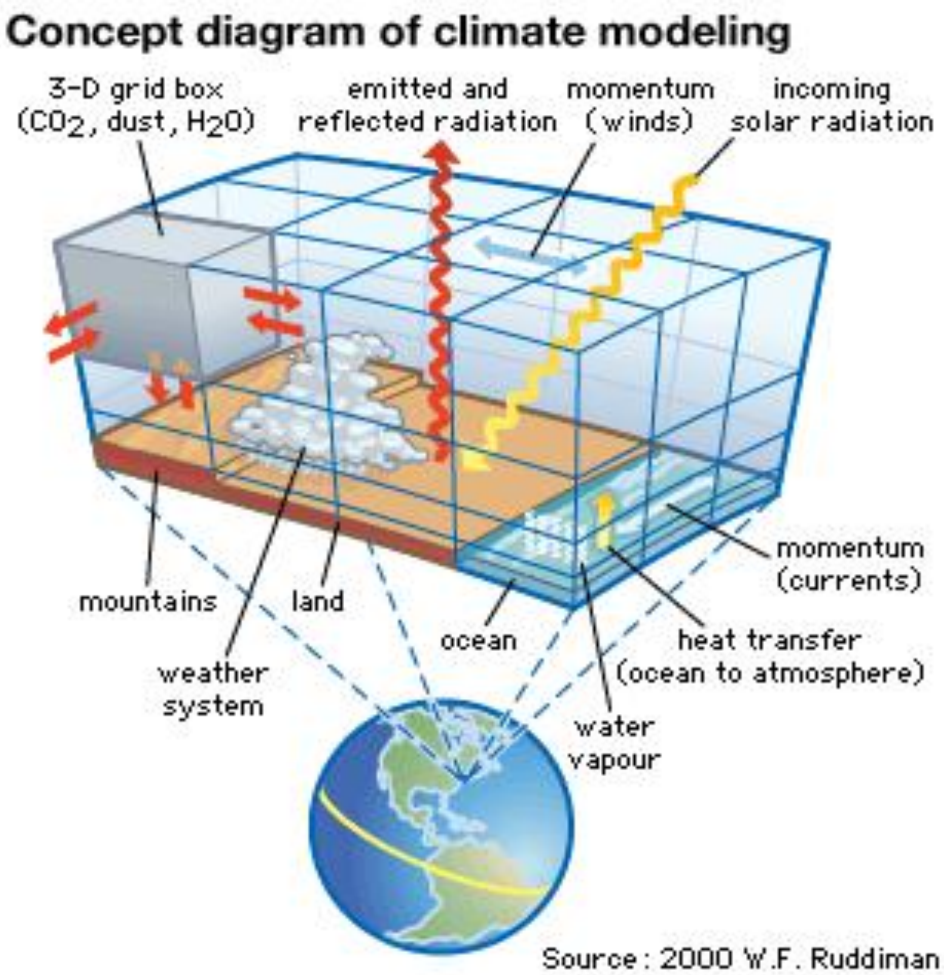
Temperature

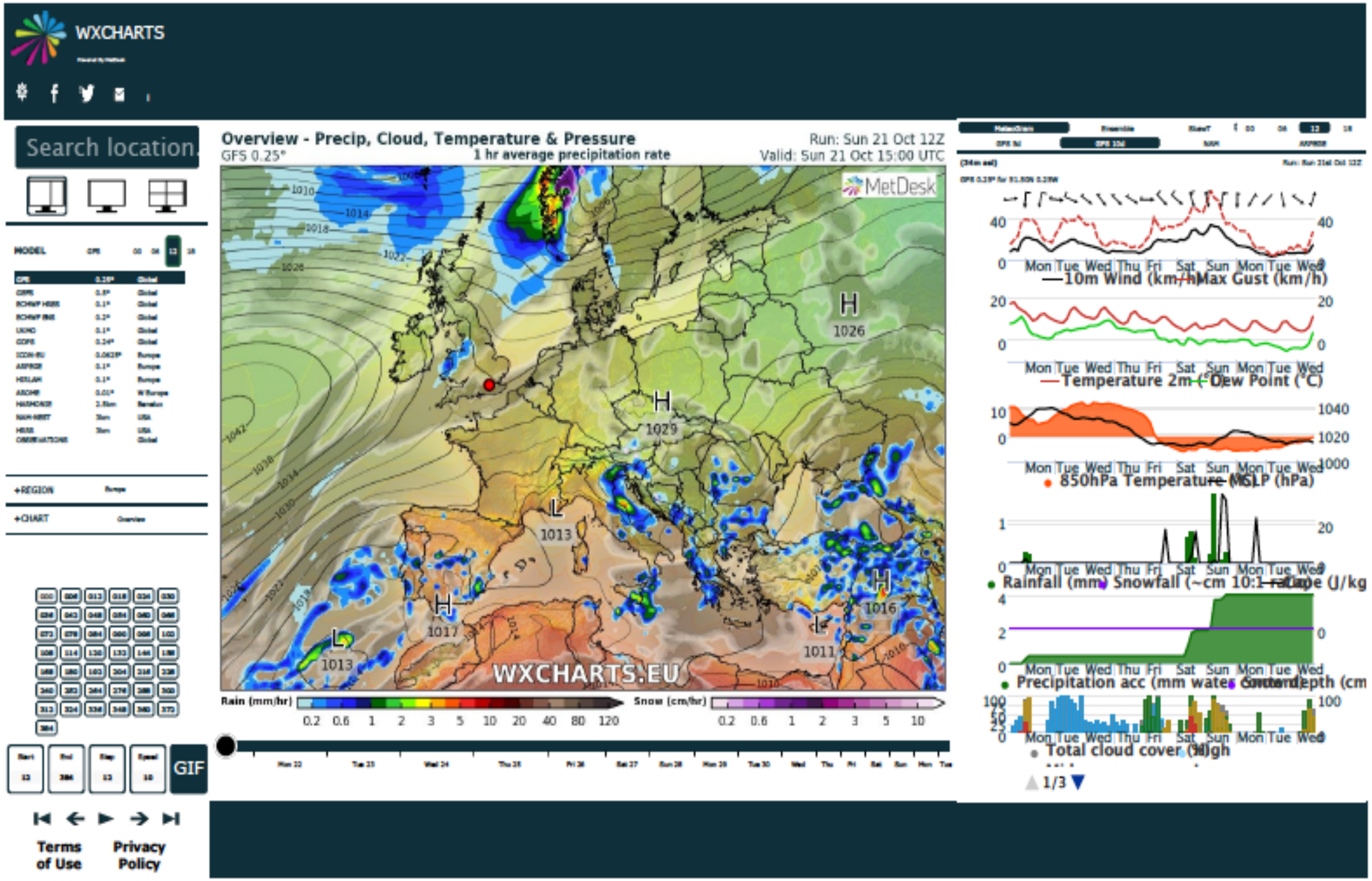
Moisture

Pressure

For climate add in ice, CO2, ocean currents, vegetation, …

Discretise and solve on a supercomputer





WXCHARTS
Search location
MODEL
+REGION
+CHART
GIF
Terms of Use
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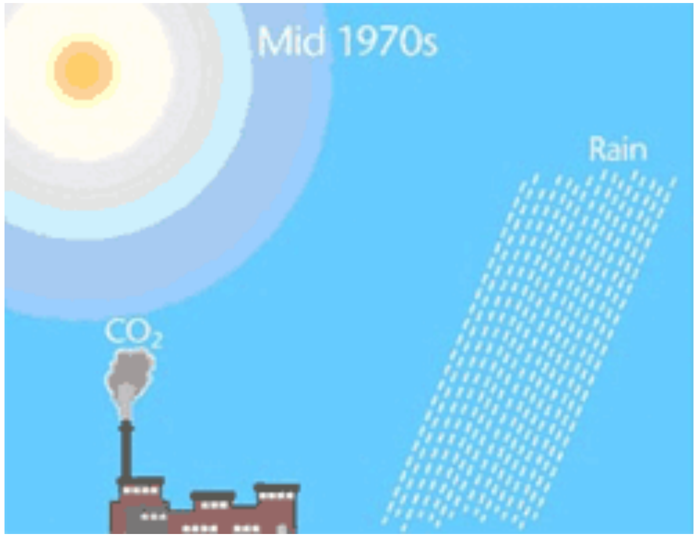
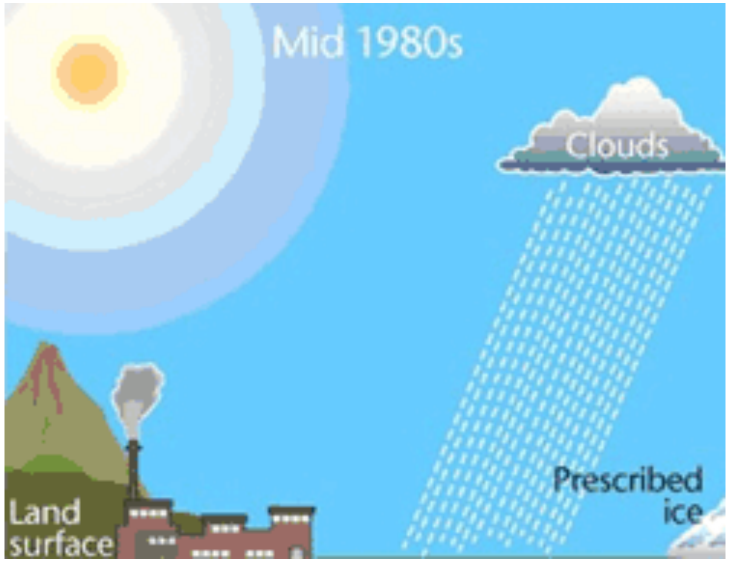
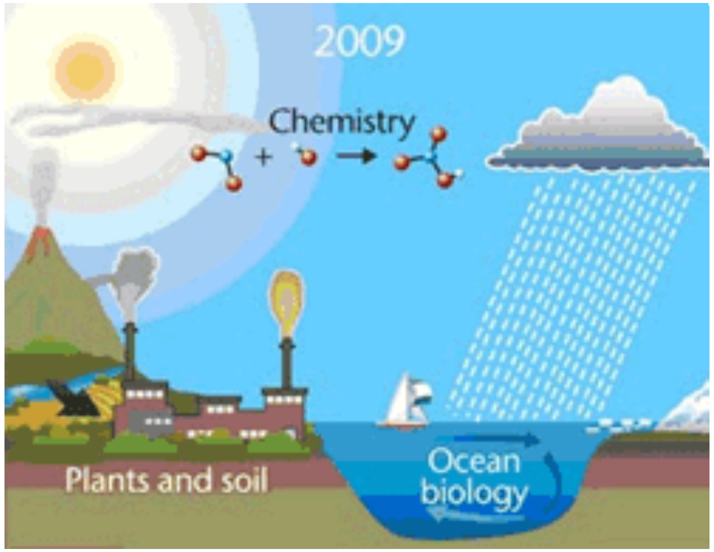
Overview - Precip, Cloud, Temperature & Pressure
GFS 0.25°
1 hr average precipitation rate
Run: Sun 21 Oct 12Z
Valid: Sun 21 Oct 15:00 UTC
MetDesk
WXCHARTS.EU
Rain (mm/hr)
0.2 0.6 1 2 3 5 10 20 40 80 120
Snow (cm/hr)
0.2 0.6 1 2 3 5 10
10m Wind (km/h)
Max Gust (km/h)
Temperature 2m (°C)
Dew Point (°C)
850hPa Temperature
MSLP (hPa)
Rainfall (mm)
Snowfall (~cm 10:1 ratio)
Cape (J/kg
Precipitation acc (mm water)
Snow depth (cm
Total cloud cover (%)
High
1/3

Climate models are constantly improving to cope with complexity





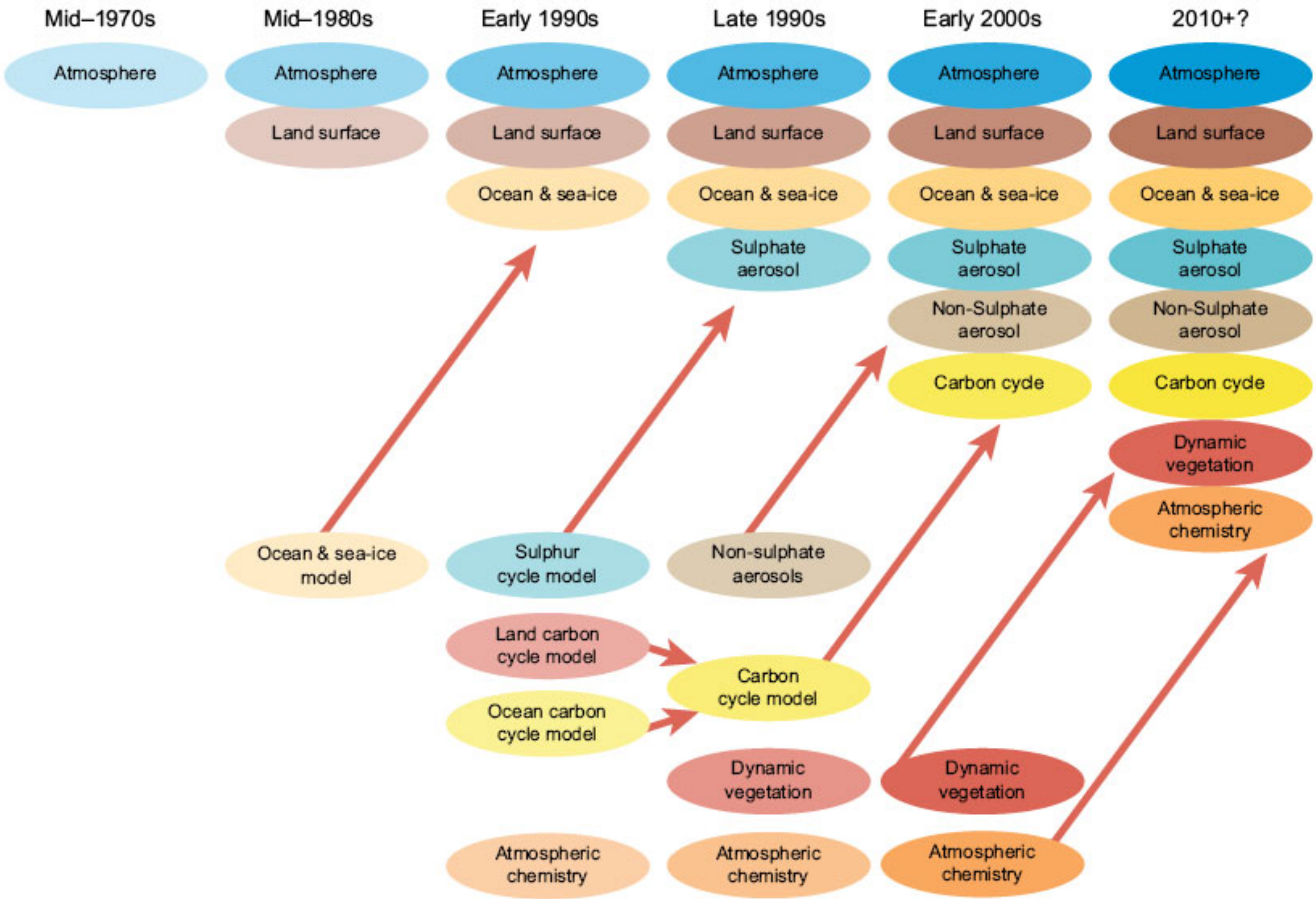


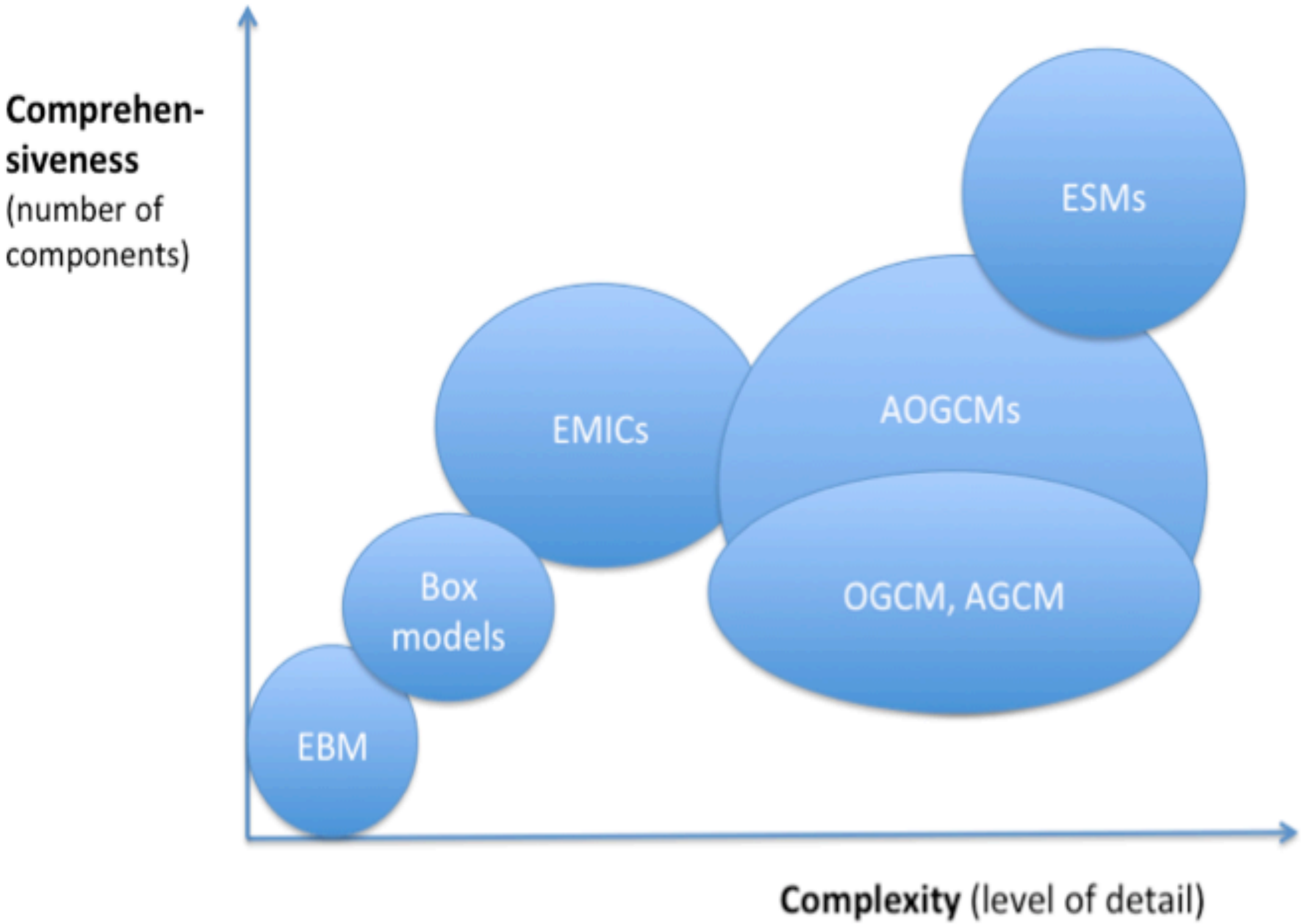
Tested by comparing with the past data

But .. Only one data set!!

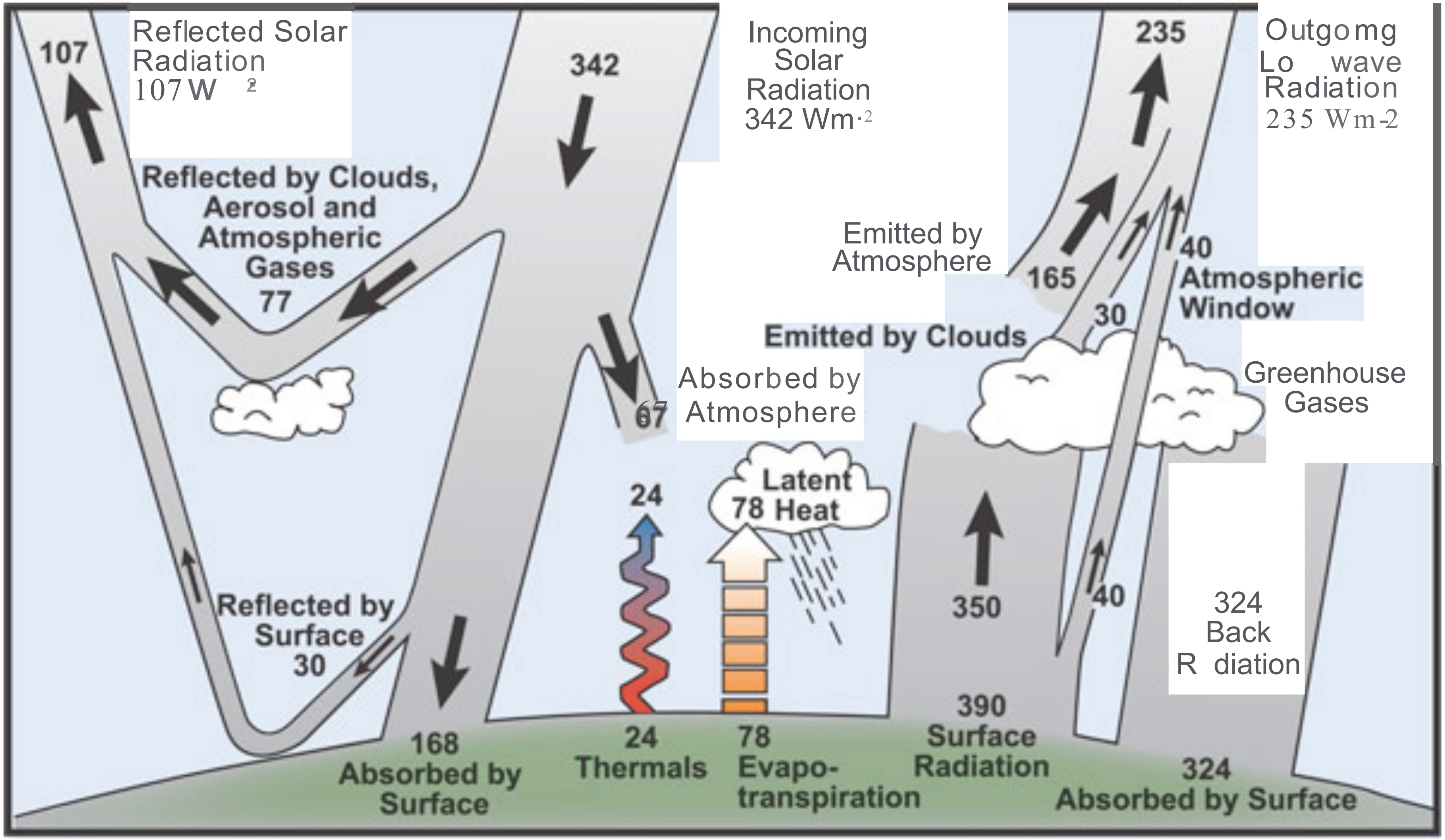
Whole range of models from simple to complex. Good, bad and ugly!

The Development of Climate Models: Past, Present and Future



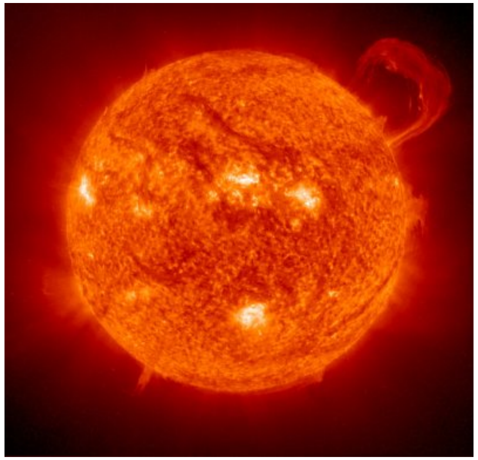

Comprehen-
siveness
(number of
components)
ESMs
EMICs
AOGCMs
OGCM, AGCM
Box
models
EBM
Complexity (level of detail)

Simplest: Energy Balance Models (EBMs)



Kiehl, J. T. and Trenberth, K. E., 1997

Simplify to a single process via averaging

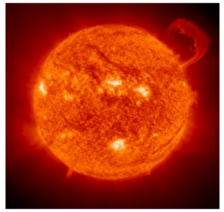


Heat into space

Heat from Sun: S

Earth’s mean temperature: T

Heat absorbed → $(1-a)S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away → $e\sigma T^4$

e emissivity: How much energy is radiated into space

Balance these to give a steady state

$$e\sigma T^4 = (1-a)S$$

If we know e, σ, a, S we can work out T

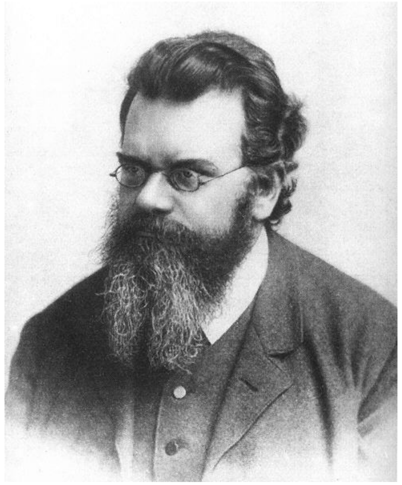
Currently

Emissivity e = 0.605,

Boltzmann $\sigma = 5.67 \times 10^{-8}$

Albedo a = 0.31,

Solar heating S = 342 W/metre^2



Work out T from the heat balance equation

$$e\sigma T^4 = (1-a)S \rightarrow T = \left(\frac{(1-a)S}{e\sigma}\right)^{1/4}$$

T = 288K

The greenhouse effect

If CO_2 increases

Then e decreases

$$T = \left(\frac{(1-a)S}{e\sigma} \right)^{1/4}$$

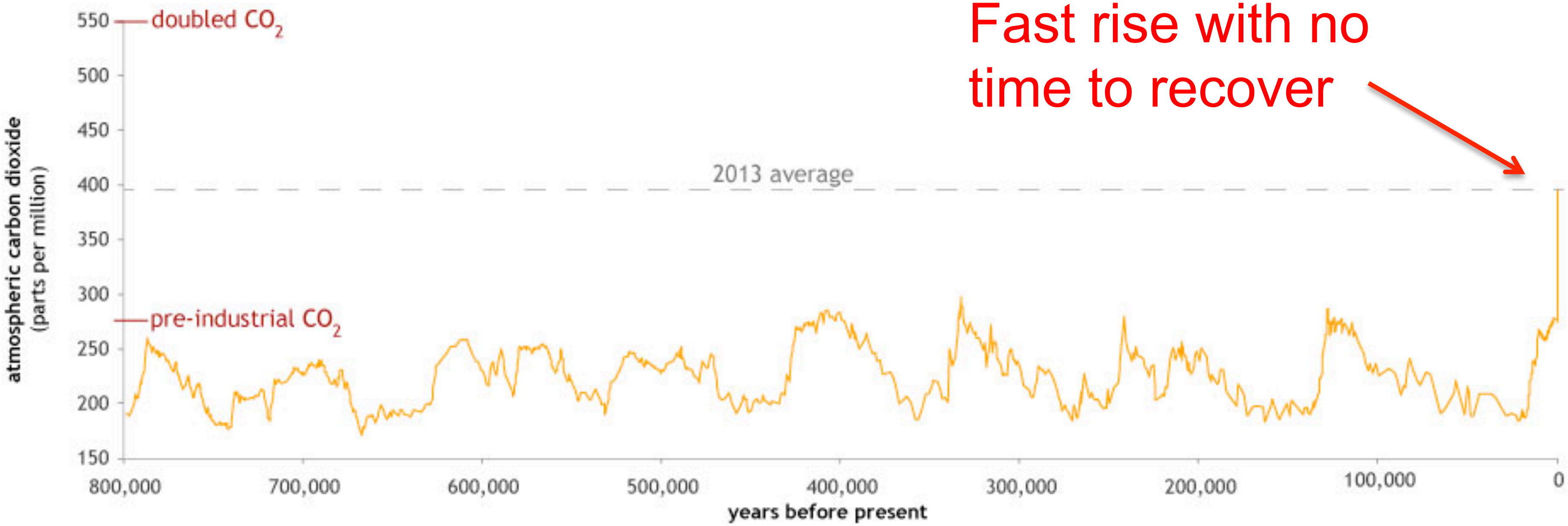
Formula tells us that T increases.

In numbers ….

Level of Carbon Dioxide (ppm)	Emissivity e_{CO2}
200	0.194
400	0.14
600	0.108
800	0.085

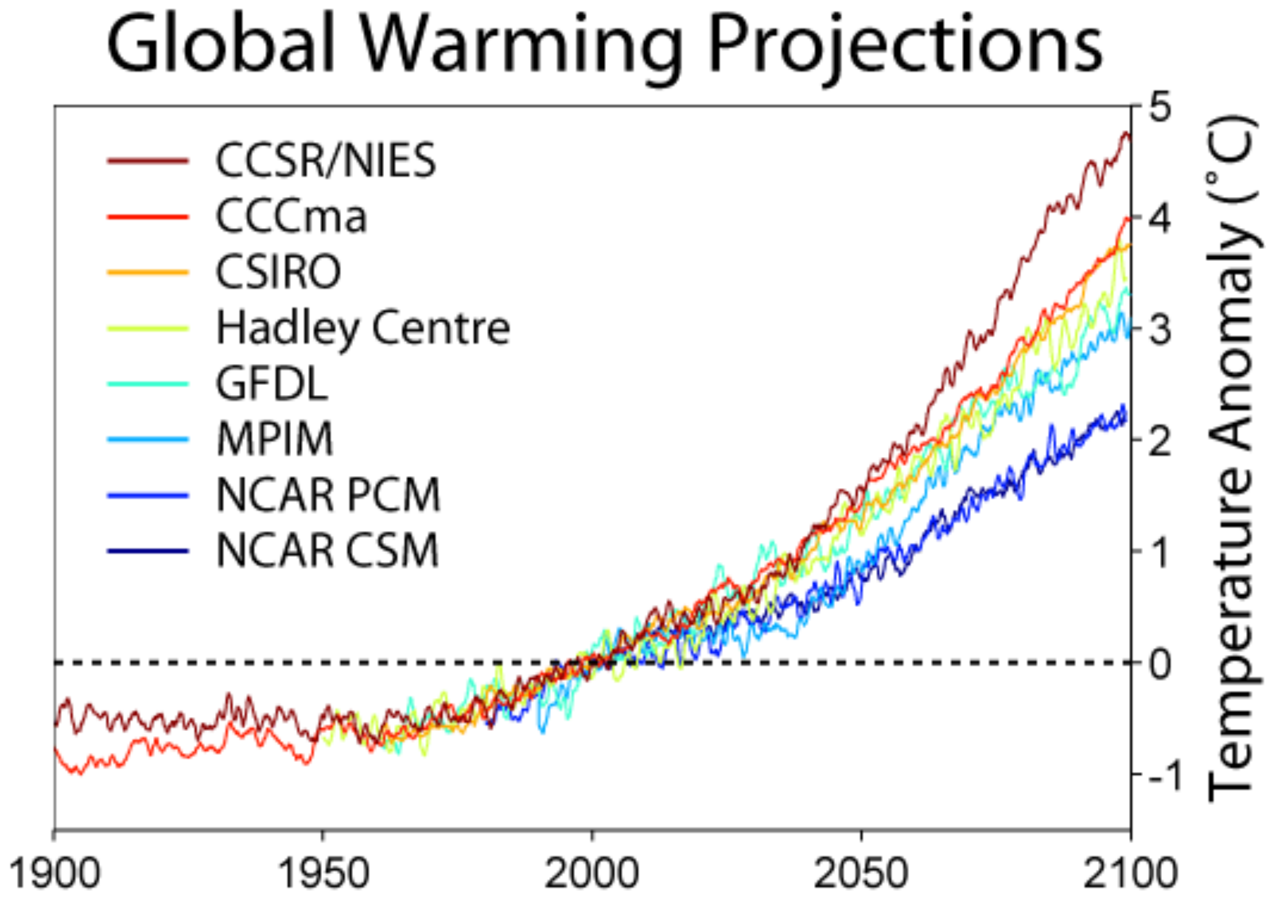
C02

Fast rise with no time to recover

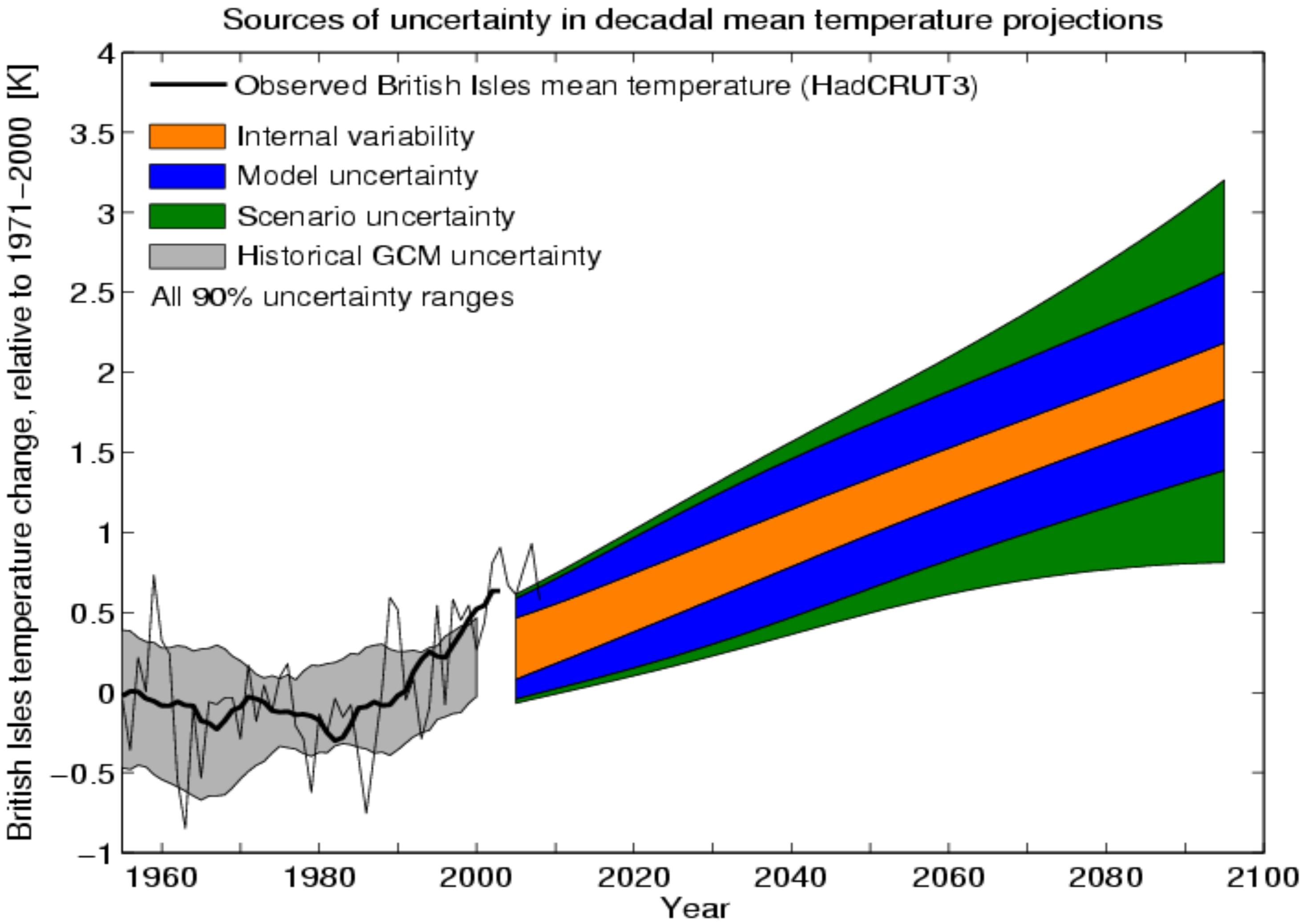


Now used to predict the future:

Gradual rise in temperature



Between a 2 and 5 degree increase by 2100



Sources of uncertainty in decadal mean temperature projections
Observed British Isles mean temperature (HadCRUT3)
Internal variability
Model uncertainty
Scenario uncertainty
Historical GCM uncertainty
All 90% uncertainty ranges
British Isles temperature change, relative to 1971−2000 [K]
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
1960
1980
2000
2020
2040
2060
2080
2100
Year

But .. Things could change more rapidly

As T increases the albedo a(T) decreases

Leads to a feedback loop

Sea ice melts

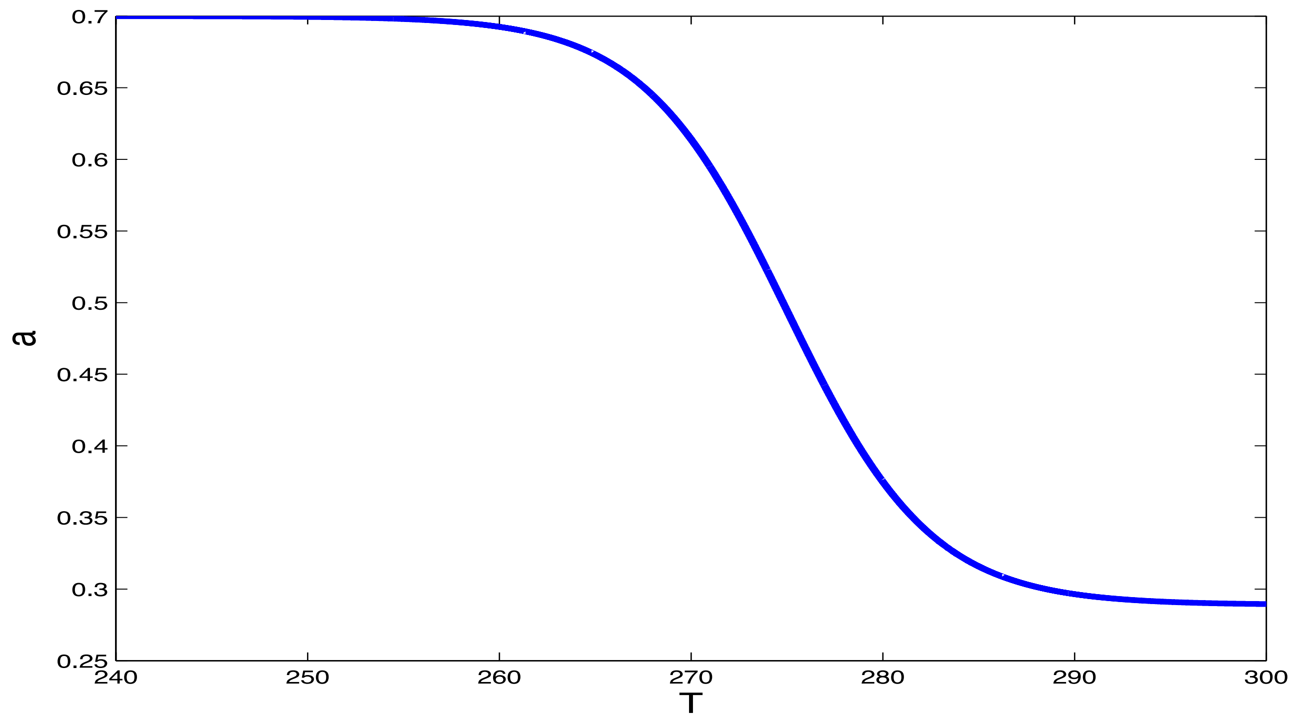
Reduced albedo

Temperature rises

This means that future temperatures may be even higher and are more sensitive to Carbon Dioxide changes

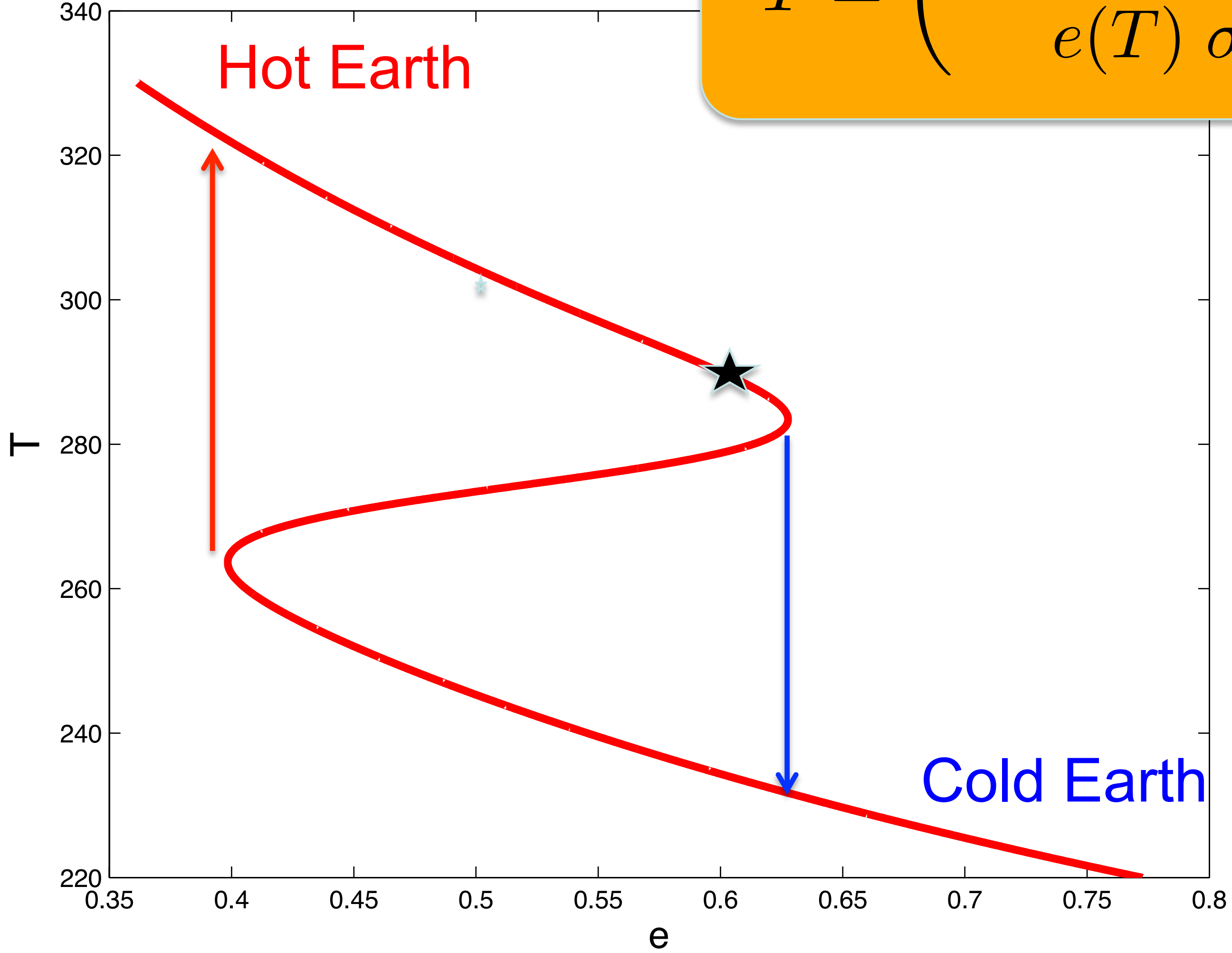
Albedo is directly linked to temperature

$$a(T) = 0.495 - 0.205 * \tanh(0.133 * (T - 275))$$

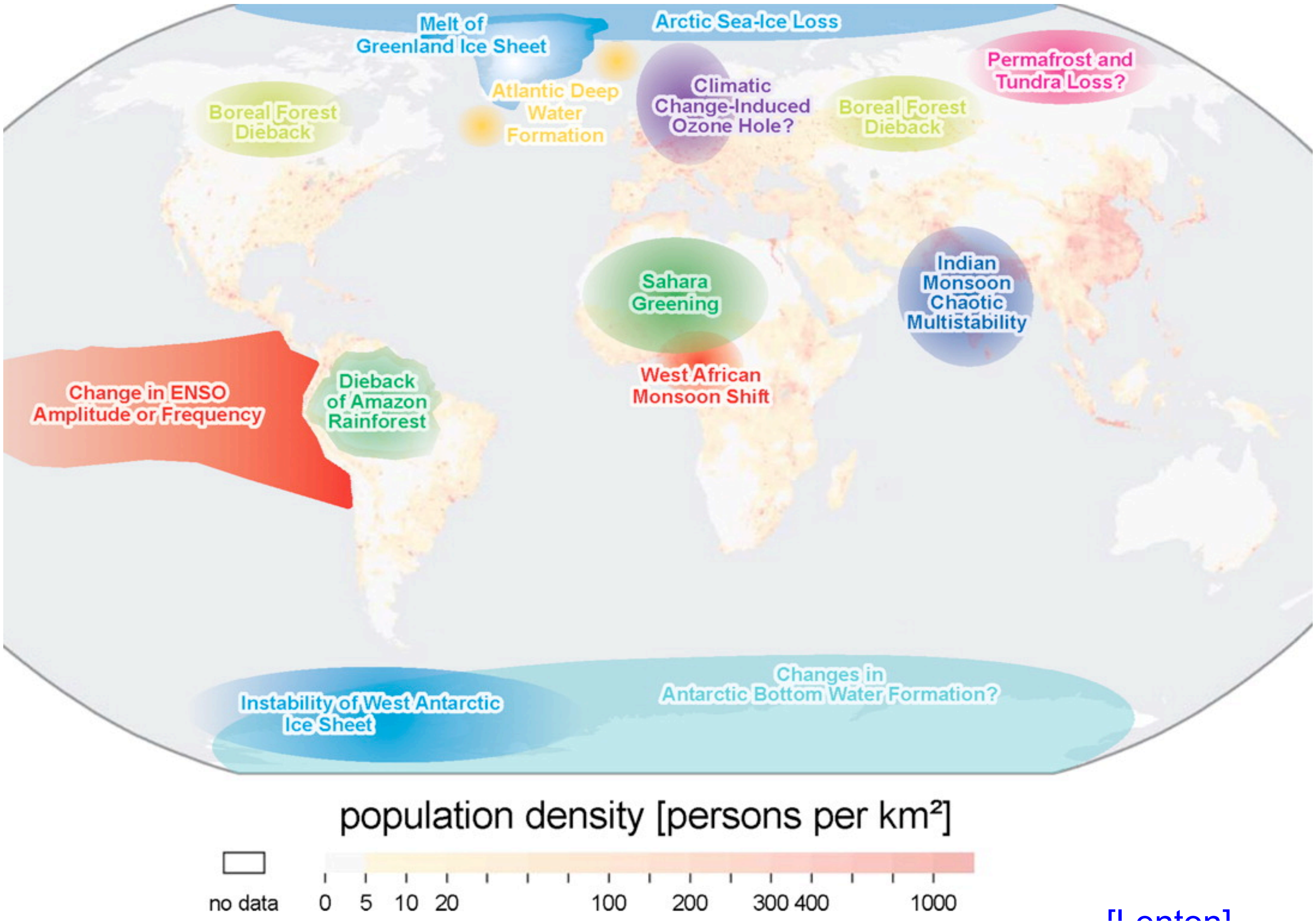


Tipping points

$$T = \left(\frac{(1 - a(T))\ S}{e(T)\ \sigma} \right)^{1/4}$$



Other Potential Tipping Points In The Climate:



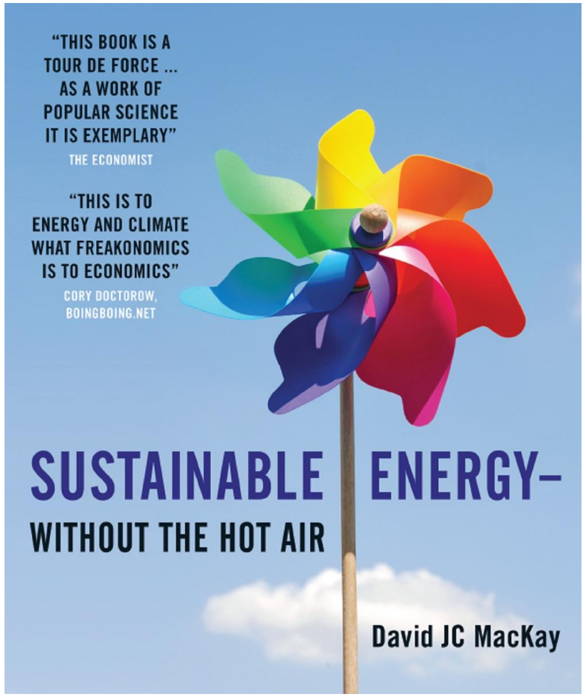
[Lenton]

So .. Are we all doomed?

Not necessarily!!

Practical ways to save the planet

- Carbon capture and storage
- Energy harvesting
- Better illumination
- Renewable energy



Conclusion

$$T = \left(\frac{(1-a)S}{e\sigma} \right)^{1/4}$$

What should a mathematician do about climate change?

- Think of ways to use less energy
- Think of better ways to produce energy
- Be aware of what is happening to our planet and the link between cause and effect
- Always use your mathematical judgment when listening to what the papers say!